

Worksheet 4

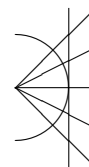
Math 202

Monday, October 18, 2021

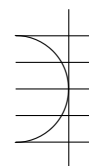
This week we're investigating *cylindrical map projections*, which map lines of latitude on the earth to horizontal lines in the plane, and lines of longitude to vertical lines.

In the equirectangular projection, a point on the surface of the earth at a latitude of α radians east and β radians north is mapped to the point (α, β) in the plane. Thus the image of the earth runs from $x = -\pi$ to $x = +\pi$ and from $y = -\pi/2$ to $y = +\pi/2$. As you move from the equator toward the poles, an east-west mile gets stretched more and more, while a north-south mile does not get stretched. Discuss this with your group, referring to your globes and your maps. Work out a formula for the east-west stretching at a given latitude β . Plug in some values of β to get a feel for the numbers.

In the central cylindrical projection, that point on the earth is instead mapped to $(\alpha, ?)$ in the plane, where $?$ is some function of β . Figure out what the function is. Now the image of the earth should run from $x = -\pi$ to $x = +\pi$ and from $y = -\infty$ to $y = +\infty$. Does this make sense in terms of the geometry of the projection, and the function you found? As you move from the equator toward the poles, an east-west mile gets stretched the same as before, but a north-south mile also gets stretched, by a different amount. Work out a formula for the north-south stretching at a given latitude β . How much does a square mile get stretched? (How much did it get stretched before?) Again, plug in some actual numbers.



Same for the Lambert cylindrical projection: What is y as a function of β ? What is the range of y -values? How much does a north-south mile get stretched, as a function of β ? How much does a square mile get stretched?



If you're making a map, you might want is angles between roads on the map to be the same as the angles between the actual roads on the earth. Discuss why none of the three projections you've looked at preserves angles, except maybe at the equator. (Consider a 1 mile by 1 mile square on the earth, and a diagonal drawn across it at a 45 degree angle, and think about the east-west and north-south stretching that you've seen.)

A map projection that preserves angles is called *conformal*. The defining properties of the Mercator projection are that it's cylindrical and conformal. How much must it stretch a north-south mile to accomplish this, as a function of the latitude β ? Is it more or less than the other projections you've looked at? How much does a square mile get stretched? Again, plug in some actual numbers.

Can you find y as a function of β to give the north-south stretching required by the Mercator projection? (When you took $y(\beta)$ and found the amount of stretching, it involved the derivative $y'(\beta)$, so going the other way will involve an integral.)

Another thing you might want a map to do is to preserve areas. You've seen that the Lambert cylindrical projection preserves areas but not angles, and that the Mercator projection preserves angles but not areas. Discuss whether any cylindrical projection can preserve both.