## Worksheet 5

## Math 202

## Monday, November 1, 2021

This week and next week we're investigating  $azimuthal^1$  map projections. When these are centered at the north or south pole, they map lines of latitude to concentric circles, and lines of longitude to lines radiating out from the center.

In the gnomonic<sup>2</sup> projection, we imagine a light shining from the center of the earth and casting a shadow onto a plane above. We've seen that the parallel at a latitude  $\beta$  has a circumference of  $2\pi R \cos \beta$ , where R is the radius of the



earth. Its shadow on the plane is a bigger circle; what is the radius? So an east-west mile get stretched by what factor? Plug in some values of  $\beta$  to get a feel for the numbers.

If you start at the north pole and travel south along a meridian, the corresponding point on the map starts at the center moves out along a line. Work out a formula for the distance from the center as a function of the latitude  $\beta$ . Do you agree that the stretching of north-south miles is given by the derivative of this function? Find the derivative and plug in some values of  $\beta$ .

Does the gnomonic projection preserve areas? Does it preserve angles?

In the *stereographic projection*, we imagine a light shining up from the south pole. Now how much does an east-west mile get stretched? A north-south mile? Don't forget to plug in some numbers. Does the projection preserve areas? Does it preserve angles?



<sup>&</sup>lt;sup>1</sup>Pronounced AZ-a-muthl.

 $<sup>^2\</sup>mathrm{Pronounced}$  no-MON-ic. I'm not making these words up.