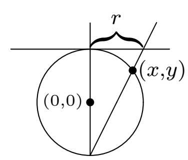
Worksheet 9

Math 202

Wednesday, November 24, 2021

Together we discussed Pythagorean triples: there's everyone's favorite $3^2 + 4^2 = 5^2$; there's $6^2 + 8^2 = 10^2$ which isn't really new; there's the less famous $5^2 + 12^2 = 13^2$; and we'd like to find more of them. We saw that this was equivalent to finding points on the unit circle $x^2 + y^2 = 1$ whose coordinates are both rational numbers.

Stereographic projection gives a correspondence between points on the unit circle and points on the horizontal line shown in this diagram:



Use similar triangles to find a formula for r in terms of x and y. What values of r correspond to $(\frac{3}{5}, \frac{4}{5})$ and $(\frac{5}{13}, \frac{12}{13})$? Is it clear from your formula that if x and y are rational numbers, then so is r?

Now go the other way and find formulas for x and y in terms of r. I recommend taking your answer to the previous question, solving for y, substituting that into $x^2 + y^2 = 1$, and solving for x. Then you can get y as well. As a sanity check, plug in the values of r that you found earlier and see if you get back $(\frac{3}{5}, \frac{4}{5})$ and $(\frac{5}{13}, \frac{12}{13})$. Is it clear from your expressions that if r is a rational number, then so are x and y?

Now plug in some new values of r to get new Pythagorean triples. Check that they really satisfy $a^2 + b^2 = c^2$. Observe that you can get infinitely many Pythagorean triples this way.

Extra: If you like programming, you could write a program to print out lots of Pythagorean triples.