

# Homework 4

Due Monday, October 28, 2019

Math 206

1. If you roll three dice, what is the probability that the sum of the numbers you get is at least 15?
  - (a) Find the answer by hand.
  - (b) Write some code to estimate the answer experimentally.
2. You have a bin with 10 red balls, 5 yellow balls, 4 blue balls, and 8 green balls. You choose four balls at random. (After choosing each ball, you keep it out and do not return it to the bin.) What is the probability that you get at least three red balls?
  - (a) Write some code to estimate the answer experimentally.
  - (b) Think about how to find the exact answer by hand.
3. What is the probability that if we flip a coin ten times, at least seven heads occur?
  - (a) Write some code to estimate the answer experimentally.
  - (b) Think about how to find the exact answer by hand.
4. How many cards do you need to choose from a standard deck in order for the probability of having at least three of a kind – that is, three of the same rank – be at least 50%? Write some code to estimate the answer experimentally.

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5. **The Birthday Problem.** You may be surprised to learn that in a room with at least 23 people, it's more likely than not that two of them will have the same birthday.
- (a) Think about how to prove this by hand. Start by ignoring leap years, so there are 365 possible birthdays. Hint: The probability that the second person has a different birthday from the first is  $364/365$ . If that happens, then the probability that the third person has a different birthday from the first two is  $363/365$ . Do you see why this is true? How should you continue?
  - (b) Write some code to carry out the following experiment: Randomly choose 23 numbers between 1 and 365. Decide whether they were all different, or whether at least two were the same. Do this over and over to find the probability, and see how it compares to 50%. Maybe try it again with 22 people, or 24.
  - (c) Does the answer change if you take leap years into account?