Math 253
Homework 6
Due Friday, February 23, 2024

For each series, decide which convergence test will be easiest to apply, then use it to decide whether the series converges or diverges. ("Easiest" is a matter of taste, so there's not necessarily a right or wrong answer to that part, but if you find yourself working very hard then consider trying another test.)

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln n}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

3. $\displaystyle\sum_{n=1}^{\infty} \ln n$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^n}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3-1}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

Taylor polynomials:

10. Find the eighth Taylor polynomial for $\sin x$: that is, find the (unique) polynomial of degree 8 whose value at zero and whose first eight derivatives at zero all agree with those of $\sin x$. If you want, put it into Desmos like we did in class; but don't turn in any graphs.

11. Find the third Taylor polynomial for $\sqrt{1+x}$. Again, put it into Desmos if you want.

Radius of convergence:

12. For which values of x does the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ converge?