Math 253
Homework 7
Due Friday, March 1, 2024

1. Find the third Taylor polynomial for $\ln\cos x$. Hint: three of the five terms are zero.

2. In lecture we found the Taylor series for $\ln(x+1)$ by taking many derivatives, plugging in $x=0$, and noticing a pattern; then we showed that this series converged for $-1<x\leq 1$. This problem will walk you through something similar with $\ln(x+2)$; the point is to see what's similar and what's different. It may be helpful to refer to your notes on what we did with $\ln(x+1)$.

   a) Let $f(x)=\ln(x+2)$. Find $f(0)$, $f'(0)$. $f''(0)$, and so on up to the fifth derivative $f^{(5)}(0)$.

   b) Find the fifth Taylor polynomial for $\ln(x+2)$.

   c) Can you spot the pattern and say what the n^th^ term of the Taylor series will be? That is, if we keep going, what will the term involving x^n look like?

   d) For which values of x does the Taylor series you found in part (c) converge? As with the examples we've done in lecture, you should start by taking absolute values and applying the ratio test, then deal with the two cases where the ratio test is inconclusive.

The next problem looks back to chapter 5, but asks you to think a little more deeply.

3. Suppose that $p\geq 0$ and $q\geq 0$. For which values of p and q does $\sum_{n=2}^{\infty}\frac{1}{n^p(\ln n)^q}$ converge? Why?

   Hint: Maybe start by messing around particular values of p and q before taking a stab at the general answer. We've seen a lot of examples with different values of p and q, sometimes using the integral test, sometimes the basic comparison test, and sometimes the limit comparison test. (But that scratch work isn't something to turn in, it's just to help you get a feeling for the question.)