

Midterm 1
Math 253
February 9, 2024

Name: _____

Each part is worth 5 points, for a total of 60 points.
You may use a hand-written sheet of notes.
Show your work where appropriate.
No calculators or cheating.

1. Find a formula for the general term a_n in the following sequences.
Indicate whether you're starting from $n=1$ or $n=0$; either choice is ok.

a) $5, -10, 15, -20, 25, \dots$

b) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \dots$

2. Suppose that $a_1=2$, and for $n \geq 2$ we have $a_n=a_{n-1}+3$.

a) Write out the first five terms of the sequence.

b) Find an explicit formula for a_n .

3. Evaluate the following limits:

a) $\lim_{n \rightarrow \infty} \frac{e^{n+1}}{e^n + 1}$

b) $\lim_{n \rightarrow \infty} \frac{n^2}{\ln n}$

4. Consider the series $\frac{1}{4} + \frac{2}{9} + \frac{3}{16} + \frac{4}{25} + \frac{5}{36} + \dots$.

a) Write it in sigma notation, that is, as $\sum_{n=1}^{\infty} (\text{something})$ or $\sum_{n=0}^{\infty} (\text{something})$.

b) Find the first three partial sums S_1, S_2, S_3 .

5. Geometric series:

a) Fill in the blank: if $|r| < 1$, then

$$1 + r + r^2 + r^3 + \dots = \underline{\hspace{4cm}}.$$

b) Does the geometric series $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \dots$ converge or diverge?
If it converges, find the limit.

6. Use the integral test to decide whether the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+5}}$

b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$