Midterm 1

Math 253

February 9, 2024 Name: Solutions

Each part is worth 5 points, for a total of 60 points.

You may use a hand-written sheet of notes.

Show your work where appropriate.

No calculators or cheating.

1. Find a formula for the general term in the following sequences.
Indicate whether you’re starting from or ; either choice is ok.

	1. If you start from , you’ll get or .

	If you start from , you’ll get or .
	2. If you start from , you’ll get .

	If you start from , you’ll get .

	You could even start from and get .
2. Suppose that , and for we have .

	1. Write out the first five terms of the sequence.

	2, 5, 8, 11, 14, 17
	2. Find an explicit formula for .

	.
3. Evaluate the following limits:
	1. We see that the limit is of the form .

	One possibility is to multiply the top and bottom by , which gives

	The other possibility is to use L’Hôpital’s rule:

	.
	2. .

	We see that the limit is of the form .

	Applying L’Hôpital’s rule (just once), we get

	.
4. Consider the series .
	1. Write it in sigma notation, that is, as or .

	Reusing the answer to problem 1a, we get

	 or or even .
	2. Find the first three partial sums .

, , .

1. Geometric series:

	1. Fill in the blank: if , then

.

* 1. Does the geometric series converge or diverge?
	If it converges, find the limit.

	We can write this as . We have ,

so the series converges to , which simplifies to .

This agrees with something we already knew: .

1. Use the integral test to decide whether the following series converge or diverge.

	1. To evaluate , which we saw on a quiz the first week, we substitute ,

	so , so , so the integral becomes

	, so the sum diverges.
	2. This appeared on the homework as §5.2 #164. To evaluate , we substitute , so , so the integral becomes

	.

	This is finite, so the sum converges.