Midterm 1

Math 253

February 9, 2024 Name: Solutions

Each part is worth 5 points, for a total of 60 points.

You may use a hand-written sheet of notes.

Show your work where appropriate.

No calculators or cheating.

1. Find a formula for the general term in the following sequences.  
   Indicate whether you’re starting from or ; either choice is ok.  
   1. If you start from , you’ll get or .  
        
      If you start from , you’ll get or .
   2. If you start from , you’ll get .  
        
      If you start from , you’ll get .  
        
      You could even start from and get .
2. Suppose that , and for we have .   
   1. Write out the first five terms of the sequence.  
        
      2, 5, 8, 11, 14, 17
   2. Find an explicit formula for .  
        
      .
3. Evaluate the following limits:
   1. We see that the limit is of the form .  
        
      One possibility is to multiply the top and bottom by , which gives  
        
        
        
      The other possibility is to use L’Hôpital’s rule:  
        
      .
   2. .  
        
      We see that the limit is of the form .  
        
      Applying L’Hôpital’s rule (just once), we get  
        
      .
4. Consider the series .
   1. Write it in sigma notation, that is, as or .  
        
      Reusing the answer to problem 1a, we get  
        
       or or even .
   2. Find the first three partial sums .

, , .

1. Geometric series:  
   1. Fill in the blank: if , then

.

* 1. Does the geometric series converge or diverge?  
     If it converges, find the limit.  
       
     We can write this as . We have ,

so the series converges to , which simplifies to .  
  
This agrees with something we already knew: .

1. Use the integral test to decide whether the following series converge or diverge.  
   1. To evaluate , which we saw on a quiz the first week, we substitute ,  
        
      so , so , so the integral becomes  
        
      , so the sum diverges.
   2. This appeared on the homework as §5.2 #164. To evaluate , we substitute , so , so the integral becomes  
        
      .  
        
      This is finite, so the sum converges.