Practice Midterm 1
Math 253
February 6, 2024

You may use a hand-written sheet of notes.
Show your work where appropriate.
No calculators or cheating.

1. Find a formula for the general term a_n in the following sequences.
   Indicate whether you're starting from $n=1$ or $n=0$; either choice is ok.

   a) 2, 5, 8, 11, 14, …

   b) $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, \ldots$

2. Suppose that $a_1 = 2$, and for $n \geq 2$ we have $a_n = 3a_{n-1}$.

   a) Write out the first five terms of the sequence.

   b) Find an explicit formula for a_n.

3. Evaluate the following limits:

a) $\lim\limits_{n \to \infty} \dfrac{n^2+2n+3}{3n^2+4n+5}$

b) $\lim\limits_{n \to \infty} \dfrac{n}{(\ln n)^2}$.

4. Consider the series $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\cdots$.

a) Write it in sigma notation, that is, as $\sum_{n=1}^{\infty}(\textit{something})$ or $\sum_{n=0}^{\infty}(\textit{something})$.

b) Find the first three partial sums S_1, S_2, S_3.

c) Does the series converge or diverge? If it converges, find the sum.
Hint: It is a geometric series, although it doesn't start from 1.

5. Consider the telescoping series $\sum_{n=1}^{\infty}\left(\sqrt{n}-\sqrt{n-1}\right)$.

a) Find the first three partial sums $S_1, S_2, S_3.$

b) Give a formula for the n^th partial sum S_n.

c) Does the series converge or diverge? If it converges, find the sum.

6. Use the integral test to decide whether the following series converge or diverge.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+5)^2}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$