Practice Midterm 1: Solutions

Math 253

February 8, 2024

1. Find a formula for the general term in the following sequences.  
   Indicate whether you’re starting from or ; either choice is ok.  
   1. 2, 5, 8, 11, 14, …  
        
      If you start from , you’ll get .  
        
      If you start from , you’ll get .
   2. , , , , , …  
        
      There are many equivalent ways to write the answer.  
        
      If you start from , you’ll get or something equivalent.  
        
      If you start from , you’ll get or something equivalent .
2. Suppose that , and for we have .   
   1. Write out the first five terms of the sequence.  
        
      2, 6, 18, 54, 162.
   2. Find an explicit formula for .
3. Evaluate the following limits:

We see that the limit is of the form .  
One possibility is to multiply the top and bottom by , which gives

.  
  
The other is to use L’Hôpital’s rule twice:

* 1. .  
       
     Again we see that the limit is of the form .  
       
     Applying L’Hôpital’s rule once, we get .  
       
     Simplifying, this becomes , which is again of the form .  
       
     Applying L’Hôpital’s rule again, we get .

1. Consider the series .
   1. Write it in sigma notation, that is, as or .  
        
      Reusing the answer to problem 1a, we get  
        
       or .
   2. Find the first three partial sums .

, , .

* 1. Does the series converge or diverge? If it converges, find the sum.  
     Hint: It is a geometric series, although it doesn’t start from 1.  
       
     We know that if then .  
     Thus .

Thus the series we’re considering can either be found as , or as .

1. Consider the telescoping series .  
   1. Find the first three partial sums

* 1. Give a formula for the nth partial sum .
  2. Does the series converge or diverge? If it converges, find the sum.  
       
     The limit of the partial sums is , so the series diverges.

1. Use the integral test to decide whether the following series converge or diverge.  
   1. To evaluate , which we saw on the very first quiz, we substitute ,  
      so , so , so the integral becomes

.  
  
This is finite, so the sum converges.

* 1. To evaluate , we substitute , so , so the integral becomes

.

Thus the sum diverges.