Practice Midterm 1: Solutions

Math 253

February 8, 2024

1. Find a formula for the general term in the following sequences.
Indicate whether you’re starting from or ; either choice is ok.

	1. 2, 5, 8, 11, 14, …

	If you start from , you’ll get .

	If you start from , you’ll get .
	2. , , , , , …

	There are many equivalent ways to write the answer.

	If you start from , you’ll get or something equivalent.

	If you start from , you’ll get or something equivalent .
2. Suppose that , and for we have .

	1. Write out the first five terms of the sequence.

	2, 6, 18, 54, 162.
	2. Find an explicit formula for .
3. Evaluate the following limits:
	1.

We see that the limit is of the form .
One possibility is to multiply the top and bottom by , which gives

.

The other is to use L’Hôpital’s rule twice:

* 1. .

	Again we see that the limit is of the form .

	Applying L’Hôpital’s rule once, we get .

	Simplifying, this becomes , which is again of the form .

	Applying L’Hôpital’s rule again, we get .
1. Consider the series .
	1. Write it in sigma notation, that is, as or .

	Reusing the answer to problem 1a, we get

	 or .
	2. Find the first three partial sums .

, , .

* 1. Does the series converge or diverge? If it converges, find the sum.
	Hint: It is a geometric series, although it doesn’t start from 1.

	We know that if then .
	Thus .

Thus the series we’re considering can either be found as , or as .

1. Consider the telescoping series .

	1. Find the first three partial sums

* 1. Give a formula for the nth partial sum .
	2. Does the series converge or diverge? If it converges, find the sum.

	The limit of the partial sums is , so the series diverges.
1. Use the integral test to decide whether the following series converge or diverge.

	1. To evaluate , which we saw on the very first quiz, we substitute ,
	so , so , so the integral becomes

.

This is finite, so the sum converges.

* 1. To evaluate , we substitute , so , so the integral becomes

.

Thus the sum diverges.