Practice Midterm 2: Solutions

Math 253

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1. Does converge or diverge, and why?  
     
   We can apply the basic comparison test. If n is big then , so , so . We know that converges by the integral test (because ), so converges as well.
2. Does converge or diverge, and why?  
     
   We want to compare to , which diverges by the integral test, but we cannot use the basic comparison test because our series is *less* than this one. But we can use the limit comparison test. The terms of the first series over those of the second are , where the second equality came from multiplying top and bottom by . Taking the limit as we get , so the limit comparison test says that either both series converge or both diverge. We have already said that diverges, so diverges as well.
3. Does converge absolutely, conditionally, or not at all?  
     
   Throwing away the signs, we get which converges by the integral test because . Thus the original series converges absolutely.
4. Does converge or diverge, and why?  
     
   We apply the ratio test, and consider  
     
   .  
     
   We know that , and similarly , so this cleans up to give  
     
   .  
     
   As , this approaches , either by applying L’Hôpital’s rule twice or by multiplying top and bottom by . Since , the series converges.
5. Find the fourth Taylor polynomial of the function .  
     
   Taking derivatives with the chain rule, we get   
   , , , .  
     
   Plugging in to f and its derivatives, we get  
   , , , , .  
     
   Thus the fourth Taylor polynomial is  
   .
6. For which values of x does the series converge?  
     
   We take absolute values and apply the ratio test, then check the cases where the ratio test is inconclusive. If we say that , then we have ,  
   where in the last step we have used the fact that . If x is fixed and then this goes to zero, which is less than 1, so the series converges converges for *any* value of x.