

Homework 2

Due Wednesday, January 25, 2017

1. Let a group G act on a set S . For a fixed $s \in S$, show that the stabilizer

$$\text{Stab}(s) = \{g \in G : g \cdot s = s\}$$

is a subgroup of G .

2. Let $G = D_4$ be the symmetry group of the square, let $r \in G$ be rotation through 90° clockwise, and let s be reflection left-to-right. In lecture we saw that

$$G = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}.$$

Decide which elements on the left are equal to which elements on the right:¹

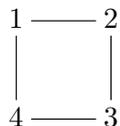
rs	sr
r^2s	sr^2
r^3s	sr^3

Do the same with D_5 , the symmetry group of the regular pentagon:

rs	sr
r^2s	sr^2
r^3s	sr^3
r^4s	sr^4

¹In the version of this homework that I handed out, there was a disastrous typo: rs^2 and rs^3 rather than r^2s and r^3s , and similarly for D_5 .

3. Label the vertices of the square as shown:



Let D_4 act on the set of 16 *ordered* pairs of vertices:

$$(1, 1) \quad (1, 2) \quad (1, 3) \quad (1, 4)$$

$$(2, 1) \quad (2, 2) \quad (2, 3) \quad (2, 4)$$

$$(3, 1) \quad (3, 2) \quad (3, 3) \quad (3, 4)$$

$$(4, 1) \quad (4, 2) \quad (4, 3) \quad (4, 4).$$

Indicate the action of r and s by drawing arrows.

There are three orbits. For each one, list the elements; then choose an element and describe its stabilizer.

4. Let G be the additive group of \mathbb{R} . Let G act on $S = \mathbb{R}^2$ by

$$\theta \cdot (x, y) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta).$$

Verify that this is a group action, i.e. that

$$0 \cdot (x, y) = (x, y)$$

and that

$$\theta \cdot (\phi \cdot (x, y)) = (\theta + \phi) \cdot (x, y).$$

Describe the orbit and stabilizer of the point $(1, 0)$. Describe all the other orbits.

5. Optional: Let $\text{GL}_n(\mathbb{R})$ act on the set of k -dimensional subspaces of \mathbb{R}^n , for a fixed $k \leq n$. Show that the action is transitive, i.e. for any two k -dimensional subspaces $W, W' \subset \mathbb{R}^n$, there is an $A \in \text{GL}_n$ such that $A \cdot W = W'$. Describe the stabilizer of the subspace spanned by the standard basis vectors e_1, e_2, \dots, e_k .