Homework 5

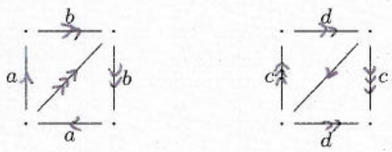
Due Monday, February 20, 2011

1. Let $X = S^2/\{(0,0,\pm 1)\}$ be the sphere with the north and south poles glued together. Compute $\pi_1(X)$, either directly using Van Kampen's theorem, or by observing that X is homotopy equivalent to a wedge sum of two spaces, or by observing that X is homotopy equivalent to a space with a disc glued along a loop.

2. Show that the groups

$$G = \langle a, b \mid abba = 1 \rangle$$
$$H = \langle c, d \mid cdcd^{-1} = 1 \rangle$$

are isomorphic. Your proof should be purely group-theoretic, but it may help to consider the following diagrams:



(Pro tip: After exhibiting a homomorphism $\varphi : G \to H$, you may be tempted to work hard showing that it is injective and surjective. It is easier just to exhibit an inverse $\psi : H \to G$.)

3. Use van Kampen's theorem to show that $\pi_1(S^n) = 1$ for $n \geq 2$. Why does the same argument not show that $\pi_1(S^1) = 1$?

4. What is one question you have about last week's lectures?