Problem Set 13

Friday, May 5

I. Problems to be graded on completion.

1. Solve the given separable differential equation with the given initial condition. Plug your solution back into the original equation to make sure that it really is a solution.

    a. $y' = y\sin(2x + 3)$, $y = 5$ when $x = 0$.

    b. $y' = \dfrac{y^2 + 1}{y + yx}$, $y = 0$ when $x = 2$.

    c. $y' = \dfrac{e^{x-y}}{1 + e^x}$, $y = 0$ when $x = 1$.

2. Solve the given inseparable differential equation with the given initial condition. Plug your solution back into the original equation to make sure that it really is a solution.

    a. $y' + y = x$, $y = 1$ when $x = 0$. You will need to know that

    $$\int xe^x \, dx = xe^x - e^x + C = (x - 1)e^x + C.$$

    Verify that this is true by taking the derivative.

    b. $xy' - 2y = x^3$, $y = 0$ when $x = 1$. Pay attention to the minus sign.

    c. $y' + y\tan x = \sec x$, $y = 2$ when $x = 0$.

    d. $y' - y\tan x = \sec x$, $y = 2$ when $x = 0$.

3. When a boat drifts in still water, the drag (i.e. acceleration) is proportional to its velocity. Suppose a boat is moving at 4 miles per hour when the motor is shut off. A minute later, it is moving at 2 miles per hour. Find an equation to describe its velocity. How far does it drift in that one minute?

4. The population of the world was approximately 4.5 billion in 1980 and 5.3 billion in 1990. From these two numbers, what would you expect the population to be now? How does this compare to actual number, 6.5 billion?

5. Over the course of a year, 100 grams of Colbalt-60 decays into 88 grams. What is the half-life of Colbalt-60? That is, how many years will it take the original 100 grams to decay into 50?

6. A cup of coffee is served at at 185°F in a room that is 65°F. Two minutes later, the temperature of the coffee has dropped to 155°F. How many more minutes will it take to cool to 105°F?

7. An advertising company is advertising a new product on TV. Suppose that the exposure to new people is proportional to the number of people who have not seen the ad, out of a total population of M viewers. Let $P(t)$ denote the number of viewers who have seen the ad at time t. No one knew about the product when the ad started running, and after 10 days, 30% of the population was aware of it. How long will it take for 90% of the population to be aware of the product?

II. Problems to be graded on correctness.

• Put a star next to what you think are the two hardest problems from Part I. I will grade those.