Midterm 2 Practice version 2 (Chapter 2)

The midterm will be in the following format: True/False, Multiple Choice, Fill in the Blank, and Open-Ended Questions. The midterm is essentially 2.5 quizzes. In general, the practice sheet is at least as hard as the midterm, which is harder than the quizzes. If you can do the practice sheet without trouble, then you are almost certainly prepared for the exam. Make sure that your studying eventually includes you being able to do problems like these without assistance from notes, textbook, or other people. If you cannot do the problems without assistance before the exam, there’s no reason to believe that when you get into the test you will do any better.

You are expected to recognize and be able to give a brief description of the following:

- Linear function
- Equation of a line
- Quadratic function
- Vertex of a parabola
- Zeros (Roots)
- Quadratic formula
- Polynomial of even/odd degree
- Horizontal/Vertical asymptote
- End behavior of polynomial/rational functions
- Graph transformations on $x^n$ ($n$ an integer)
- Slope of a linear function
- Parallel/Perpendicular lines
- Polynomial
- Completing the square
- Vertex form of a quadratic function
- Rational function
- Graphs of $x, x^2, x^3, x^4, \frac{1}{x}, \frac{1}{x^2}$
- Local extrema
1. For each of the following functions find the domain, roots, \( y \)-intercept (if it exists) and graph the function.

(a) \(-2x^4 + 2x^3 + \frac{7}{2}x^2 - 2x - \frac{3}{2}\)
(b) \(x^4 + \frac{16}{3}x^3 + \frac{17}{3}x^2 - \frac{8}{3}x - \frac{4}{3}\)
(c) \(\frac{x^3 - 4x^2 + x + 6}{x^3 + x^2 - 12x}\)
(d) \(\frac{x^4 - \frac{5}{2}x^3 + \frac{3}{4}x^2 + \frac{5}{8}x - \frac{1}{4}}{4x^3 + 10x^2 - 6x}\)

2. For the quadratic function \(f(x) = 2x^2 - 7x - 4\), find ...

(a) The equation for \(f\) written in the form \(k(x + t)^2 + r\).
(b) The coordinates of the vertex.
(c) The \(y\)-intercept.
(d) The \(x\)-intercept(s).

3. Find the rule of the linear function whose graph contains the point \((-5, 20)\) and is...

(a) ... parallel to the line through the points \((5, 6)\) and \((17, -2)\).
(b) ... perpendicular to the line through the points \((-22, 8)\) and \((-7, 68)\).

4. Write the equation of a polynomial function \(p\) having only roots \(-4, -2, 1,\) and \(3\), and with the given characteristics. There are many possible solutions.

(a) As \(x \to \pm\infty\), \(f(x) \to \infty\) and \(\deg p > 6\).
(b) \(p(0) = -24\) and \(p\) can have at least five local extrema.
(c) As \(x \to \pm\infty\), \(f(x) \to \mp\infty\), and all non-constant factors of \(p\) correspond to real roots.
(d) \(p\) has leading coefficient \(\pi\) and two non-constant factors that do not correspond to real roots.

5. Find all possible values of \(w\) such that \(x - 2\) is a factor of \(x^4 + w^2x^2 - 3wx - 17\).

6. i) Identify the basic function \(f\) upon which \(g(x)\) is based. ii) List the graph transformations necessary to change the graph of \(f\) into the graph of \(g\). iii) Write the equations of any asymptotes of \(g\).

(a) \(g(x) = \frac{2}{-x - 3} + 1\)
(b) \(g(x) = \frac{3}{4}x^4 + \frac{2}{3}\)
(c) \(g(x) = -\frac{1}{(\frac{4}{3}x + 2)^2}\)
(d) \(g(x) = -0.2x^3 - 6\)

7. Find all values of \(q\) such that the slope of the line through the points \((3 - q^2, -5)\) and \((2, q)\) is equal to 1.
8. Define functions \( f(x) = \frac{x^2 + 5}{x - 5}, \ g(x) = x^4 - 2x^2 + 1, \) and \( h(x) = 2x^3 - 3x^2 + 5. \) Find each of the following and simplify your answer where appropriate.

(a) \((f + h)(-1)\)
(b) \(\left(\frac{h}{g}\right)(-2)\)
(c) \(\text{deg}(g \circ h)\)
(d) \(\text{deg}(hg^2)\)
(e) \(\frac{f(c - 3) - f(3)}{c}\)
(f) \(\frac{g(a + 2) - g(2 - a)}{2a}\)

(g) The equations of any asymptotes of \(\left(\frac{h}{g}\right)(x)\)
(h) The equations of any asymptotes of \(\left(\frac{g}{h}\right)(x)\) (Hint: \(2x^3 - 3x^2 + 5 = (x + 1)(2x^2 - 5x + 5)\))

9. Given the complete graph of the polynomial function \( f(x) \) shown below, answer the following questions.

(a) Is the degree of \( f(x) \) even or odd?
(b) Is the leading coefficient positive or negative?
(c) What are the zeros of \( f(x) \)?
(d) What is its smallest possible degree?
(e) If \( f \) is of the smallest possible degree and \( f(-2) = -8.64 \), find the equation of \( f \). (Hint: the factor corresponding to the root 4 is repeated)

10. The height \( h \) (in feet) of an object that is traveling vertically at time \( t \) (in seconds) is given by

\[ h = -16t^2 + v_0t + h_0, \]

where \( h_0 \) is the initial height and \( v_0 \) is the initial velocity (a positive initial velocity means the object begins heading upward).

A penny is tossed downward at 25 feet/second from the top of the Empire State Building (1250 feet high). How long does it take to reach the ground?
11. The profit (in dollars) for selling $x$ baby strollers each month is given by $p(x) = 1600x - 4x^2 - 50000$. How many strollers does the company need to sell each month in order to maximize profit? What is that maximum profit?

12. The survival rate $s$ of seedlings in the vicinity of a parent tree is given by $S(x) = \frac{0.5x}{1 + 0.4x^2}$ where $x$ is the distance from the seedling to the parent tree (in meters). (The model is most accurate when $0 < x \leq 10$).

(a) At what distance(s) does the seedling survival rate fall to 0.15?

(b) What is the exact maximum value of $S$? Interpret your answer.

True/False + Explanation (Decide whether each of the following statements is always true or sometimes false. Give a full justification of your decision including graphs, examples, and calculations where appropriate. Make sure you conclude your answer in words, though.):

1. $x - 1$ is a factor of $g(x) = 2x^{67} - 4x^{13} + x - 6$.

2. A rational function has a vertical asymptote at $x = c$ every time $c$ is a zero of the denominator.

3. The x-value of the vertex of a parabola is the midpoint of the x-intercepts (if they exist) of the parabola.

4. Graphs of rational functions contain no gaps, cusps, or asymptotes.

5. A polynomial of odd degree is also an odd function.

Multiple Choice (Choose the best answer from among the choices given)

1. The graph of $f(x) = \frac{x - 2}{x^2 - 4}$ has...

   (a) ... exactly one vertical asymptote.
   (b) ... exactly two vertical asymptotes.
   (c) ... exactly one horizontal asymptote.
   (d) ... only (b) and (c) true.
   (e) ... only (a) and (c) true.

2. For a polynomial function $t(x)$, we find that $t(-3) = 0$. This fact implies that...

   (a) ... $x + 3$ is a root of $t(x)$.
   (b) ... $x + 3$ is a factor of $t(x)$.
   (c) ... $x - 3$ is a root of $t(x)$.
   (d) ... $x - 3$ is a factor of $t(x)$. 