Math 232, Discrete Mathematics II, Winter 2023, Professor Arkady Berenstein

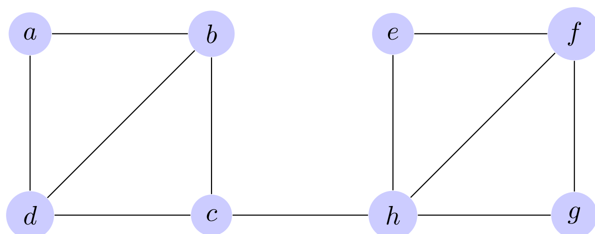
PRACTICE PROBLEMS FOR FINAL EXAM

1. Solve the following recurrence relations:
(a) $a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 0$, $a_1 = 1$.
(b) $a_n = 2a_{n-1} - 3a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 1$.
(c) $a_n = -3a_{n-1} + 10a_{n-2}$, $n \geq 2$, $a_0 = 2, a_1 = 1$.

2. Let A_n be the set of binary strings of length n which do not contain the string 001. Find and solve a recurrence relation for $a_n = |A_n|$.

3. How many full binary trees are there on $2n + 1$ vertices?

4. Let G be the undirected graph given below.



(a) Find the number of simple paths from a to b.
(b) Find all cycles starting at a.
(c) Determine number of vertices, edges, and regions and show that your answers satisfy Euler's Theorem.
(d) Find a dual graph.

5. (a) How many edges does the graph K_9 have?
(b) Find the maximum length of a cycle in K_9.
(c) Find the maximum length of an open simple path in K_9.

6. For which n does the complete graph K_n admit an Euler cycle?

7. For which n does the complete bipartite graph $K_{n,n}$ admit an Euler cycle?

8. Let $G = (V, E)$ be a loop-free connected graph with $V = \{v_1, v_2, \ldots, v_n\}$, where $n \geq 2$, $deg(v_1) = 1$ and $deg(v_j) \geq 2$ for all $2 \leq j \leq n$. Prove that G must have a cycle.

9. Find the number of distinct Hamilton cycles in the complete bipartite graph $K_{n,n}$, where $n \geq 2$.

10. Write the expression $(x + 1)(x^2 - x + 1) - (x^3 + 1)$ in Polish notation, using a rooted tree.

11. (a) Find a rooted binary tree corresponding to the algebraic expression $((3 + 1/x) * y^5))/(z/7)$.
(b) Find the preorder traversal.
(c) Find the postorder traversal.
(d) Find the inorder traversal.

12. Let $G = (V, E)$ be a loop-free undirected graph. Prove that if G contains no cycle of odd length, then G is bipartite.

13. Find all loop-free undirected connected graphs with five vertices up to a graph isomorphism. How many of these have no pendant vertices?

14. Let G_n be a graph which is obtained from the complete graph K_n by deleting one edge. Determine the chromatic and the chromatic number χ_{G_n}.

15. Determine whether the check digit of the ISBN-10 for the eighth edition of Discrete Mathematics and Its Applications was computed correctly by the publisher.

16. Find the smallest relation containing the relation $\{(1, 2), (1, 4), (3, 3), (4, 1)\}$ that is
(a) reflexive and transitive.
(b) symmetric and transitive.
(c) reflexive, symmetric, and transitive.