Math 232, Discrete Mathematics II, Winter 2023, Professor Arkady Berenstein

Midterm 1 will be based on sections 8.1, 8.2, 10.1-10.5, and 10.7.

PRACTICE PROBLEMS FOR MIDTERM 1

1. Solve the following recurrence relations:
(a) $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 1$.
(b) $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$, $a_0 = 0$, $a_1 = 1$.
(c) $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = -3$.
(d) $a_n = 2a_{n-1} + 2a_{n-2}$, $n \geq 2$, $a_0 = 0$, $a_1 = 1$.

2. Find the number a_n of strings of length n over the alphabet $\mathbf{A} = \{0, 01, 111\}$.

3. Find a connected graph G with 10 vertices, where removing any edge of G results in a graph with an isolated vertex.

4. A graph $G = (V, E)$ with 21 edges has seven vertices of degree 1, three of degree 2, seven of degree 3 and the rest of degree 4. How many vertices does it have?

5. Find all loop-free undirected graphs with four vertices up to a graph isomorphism. How many of these have no pendant vertices?

6. If $G = (V, E)$ is a connected graph with $|E| = 17$ and $\deg(v) \geq 3$ for all $v \in V$, what is the maximum number $|V|$ of vertices? Then draw such a graph to demonstrate that it exists.

7. Let $G = (V, E)$ be a loop-free connected 4-regular planar graph. If $|E| = 16$, how many regions are there in a planar depiction of G?

8. Which, if any, of the pairs of graphs shown are isomorphic? Justify your answer by describing an isomorphism or explaining why one does not exist.