

Math 253, Calculus III, Spring 2024

MIDTERM 1 STUDY GUIDE

Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

1. All questions from Quiz 1.
2. All assigned homework problems (Sections 5.1—5.4).
3. Determine whether the sequence converges or diverges. If converges, compute the limit.

a. $a_n = \frac{3 + n^3}{1 + 3n^3}$

b. $a_n = \frac{n^3}{1 + n^2}$

c. $a_n = \frac{2^{3n+1}}{3^{2n+5}}$

d. $a_n = \frac{\ln(n)}{\sqrt{n}}$

e. $a_n = \frac{n \cos(n)}{1 + n^3}$

f. $a_n = \frac{(-1)^n 3^n}{2^{2n}}$

g. $a_n = \ln(3n^3 + 1) - \ln(n^3 + 1)$

h. $a_n = \frac{1}{n(\sqrt{n+1} - \sqrt{n-1})}$

i. $a_n = \frac{1}{\sqrt{n^2 + n} - \sqrt{n^2 - n}}$

4. Express the repeating decimal as a fraction.
 - a. 2.121212...
 - b. 3.131313...
 - c. 3.141414...
 - d. 2.717171...
 - e. 9.999999...

5. Find the sum of the series.

a. $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{3^{2n+1}}$

b. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{2n+1}}$

c. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^{2n+1}}$, where $-8 < x < 10$.

d. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

e. $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$

6. Use **any** test to determine whether the series converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{n}{1 + 2n^3}$

b. $\sum_{n=1}^{\infty} \frac{n^3}{1 + 2n^4}$

c. $\sum_{n=1}^{\infty} \frac{n^4}{4^n}$

d. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

e. $\sum_{n=1}^{\infty} \frac{2^n}{2^n + 3^n}$

f. $\sum_{n=1}^{\infty} \frac{3^n}{2^n + 3^n}$