

Math 253, Calculus III, Winter 2024

MIDTERM 2 STUDY GUIDE

Here are some good review questions. The actual exam might be of a different format, but these will help you understand the concepts covered on the exam. Try to do as many of these as you can without looking in your notes or book for guidance.

1. All questions from Quiz 2.
2. All assigned homework problems (Sections 5.5—6.2).
3. Determine whether each of the following series is **absolutely convergent**, **conditionally convergent**, or **diverges**. State the test you are using and show all details of the test.

a.
$$\sum_{k=1}^{\infty} \frac{2k^2 - 1}{k^2 3^k}$$

b.
$$\sum_{k=0}^{\infty} \frac{2}{(-3)^k}$$

c.
$$\sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{k \ln k}$$

d.
$$\sum_{k=2}^{\infty} \frac{10k^2}{k^3 - 1}$$

e.
$$\sum_{k=1}^{\infty} \frac{5}{4^k + 3}$$

f.
$$\sum_{k=1}^{\infty} \frac{k^{10} 2^k}{k!}$$

g.
$$\frac{1}{4!} - \frac{4}{5!} + \frac{9}{6!} - \frac{16}{7!} + \dots$$

h.
$$\sum_{k=0}^{\infty} \left(\frac{3}{2}\right)^k$$

i.
$$\sum_{k=2}^{\infty} \frac{1}{k \sqrt{\ln k}}$$

j.
$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$$

k.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

4. Determine the radius and the interval of convergence of the power series.

a.
$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$$

b.
$$\sum_{k=0}^{\infty} \frac{(x+1)^k}{3^k}$$

c.
$$\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 2^n}$$

d.
$$\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$$

e.
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt{n}}$$

f.
$$\sum_{n=0}^{\infty} \frac{3^n (x+5)^n}{(n+1)!}$$

g.
$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} (x+1)^n}{2^n}$$

h.
$$\sum_{n=0}^{\infty} \frac{n! \cdot x^n}{e^{n+2}}$$

5. Write a power series representation for the function and determine the radius and the interval of convergence.

a.
$$f(x) = \frac{1}{1+2x}$$

b.
$$f(x) = \frac{1}{(1+2x)^2}$$

c.
$$f(x) = \ln(1+2x)$$

d.
$$f(x) = \frac{x}{9-x^2}$$

e.
$$f(x) = \ln(9-x^2)$$

f.
$$f(x) = e^{-x}$$

g.
$$f(x) = e^{-5x}$$

h.
$$f(x) = e^{-5x^2}$$

i.
$$f(x) = \arctan(2x^3)$$

6. Use a power series to calculate the indefinite and definite integrals

a. $\int \frac{x - \sin(x)}{x^3} dx$

b. $\int_0^1 \frac{x - \sin(x)}{x^3} dx$

c. $\int \frac{1 - \cos(x)}{x^2} dx$

d. $\int_0^1 \frac{1 - \cos(x)}{x^2} dx$

e. $\int \frac{x - \arctan(x)}{x^2} dx$

f. $\int_0^1 \frac{x - \arctan(x)}{x^2} dx$

g. $\int \sin(x^3) dx$

h. $\int_0^1 \sin(x^3) dx$

i. $\int e^{-x^2} dx$

k. $\int_0^1 e^{-x^2} dx$