

## MATH 341, REVIEW FOR FINAL EXAM

The exam will cover the material we have discussed in class and studied in homework, from Sections 2.3, 2.5, 2.8, 2.9, 3.1-3.3, 4.1-4.6. The following list points out the most important definitions and theorems.

### Definitions:

$LU$  factorization.  
Subspace, null space, column space.  
Linear transformation.  
Linear independence, linear dependence, linear dependence relation (for vector spaces).  
Spanning set, basis, coordinate vector.  
Vector spaces, subspaces, dimension.  
Rank of a matrix and of a linear transformation.

### Theorems:

Chapter 2:

The Invertible Matrix Theorem (including new statements in Sections 3.2 ).  
Theorem 10 (Column-Row expansion of  $AB$ ).

Chapter 3:

Theorems 2, 3, 4, 5, 6 (properties of determinants).  
The formula for  $\det A$  as a product of pivots.  
Theorem 7 (Cramer's Rule).  
Theorems 9 and 10 (determinants as area or volume).

Chapter 4:

Theorems 1-3.  
Theorem 7 (Unique Representation Theorem),  
Theorem 8 (Coordinate systems).  
Theorems 9, 10, 12 (Basis Theorems).  
Know the proofs of Theorems 2 and 3.

### Important Skills:

Write  $AB$  using a column-row expansion.  
Construct an  $LU$  factorization of a matrix, use an  $LU$  factorization to solve  $Ax = b$ .  
Compute a  $3 \times 3$  or triangular determinant.  
Compute a determinant using Theorems 3, 4, 5, 6.  
Solve a  $2 \times 2$  or  $3 \times 3$  system using Cramer's Rule.  
Determine if a set of vectors spans (or is a basis for)  $\mathbb{R}^n$ .  
Determine if a set of polynomials spans (or is a basis for)  $\mathbb{P}_n$ .  
Determine if a set is a subspace (using Theorems 1, 2, or 3 in Chapter 4).  
Determine if a vector is in  $Nul A$  or in  $Col A$ .  
Find a nonzero vector in  $Nul A$  or  $Col A$ .  
Determine if a set is a basis for a subspace.  
Find a basis for  $Nul A$  or  $Col A$ , or for other subspaces of  $\mathbb{R}^n$  or of  $\mathbb{P}_n$ .  
Find the coordinate vector of a vector relative to a basis.  
Use coordinate vectors to check if a set is linearly independent.  
Find the dimension of a subspace.