Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case).

a. Every matrix is row equivalent to a unique matrix in echelon form.

b. Any system of \( n \) linear equations in \( n \) variables has at most \( n \) solutions.

c. If a system of linear equations has two different solutions, it must have infinitely many solutions.

d. If a system of linear equations has no free variables, then it has a unique solution.

e. If an augmented matrix \( [A \ b] \) is transformed into \( [C \ d] \) by elementary row operations, then the equations \( Ax = b \) and \( Cx = d \) have exactly the same solution sets.

f. If a system \( Ax = b \) has more than one solution, then so does the system \( Ax = 0 \).

g. If \( A \) is an \( m \times n \) matrix and the equation \( Ax = b \) is consistent for some \( b \), then the columns of \( A \) span \( \mathbb{R}^m \).

h. If an augmented matrix \( [A \ b] \) can be transformed by elementary row operations into reduced echelon form, then the equation \( Ax = b \) is consistent.

i. If matrices \( A \) and \( B \) are row equivalent, they have the same reduced echelon form.

j. The equation \( Ax = 0 \) has the trivial solution if and only if there are no free variables.

k. If \( A \) is an \( m \times n \) matrix and the equation \( Ax = b \) is consistent for every \( b \) in \( \mathbb{R}^m \), then \( A \) has \( m \) pivot columns.

l. If an \( m \times n \) matrix \( A \) has a pivot position in every row, then the equation \( Ax \) has a unique solution for each \( b \) in \( \mathbb{R}^m \).

m. If an \( n \times n \) matrix \( A \) has \( n \) pivot positions, then the reduced echelon form of \( A \) is the \( n \times n \) identity matrix.

n. If \( 3 \times 3 \) matrices \( A \) and \( B \) each have three pivot positions, then \( A \) can be transformed into \( B \) by elementary row operations.

o. If \( A \) is an \( m \times n \) matrix, if the equation \( Ax = b \) has at least two different solutions, and if the equation \( Ax = c \) is consistent, then the equation \( Ax = c \) has many solutions.

p. If \( A \) and \( B \) are row equivalent \( m \times n \) matrices and if the columns of \( A \) span \( \mathbb{R}^m \), then so do the columns of \( B \).

q. If none of the vectors in the set \( S = \{v_1, v_2, v_3\} \) in \( \mathbb{R}^3 \) is a multiple of one of the other vectors, then \( S \) is linearly independent.

r. If \( \{u, v, w\} \) is linearly independent, then \( u, v, \) and \( w \) are not in \( \mathbb{R}^2 \).

s. In some cases, it is possible for four vectors to span \( \mathbb{R}^3 \).  

t. If \( u \) and \( v \) are in \( \mathbb{R}^n \), then \( -u \) is in \( \text{Span}\{u, v\} \).

u. If \( u, v, \) and \( w \) are nonzero vectors in \( \mathbb{R}^2 \), then \( w \) is a linear combination of \( u \) and \( v \).

v. If \( w \) is a linear combination of \( u \) and \( v \) in \( \mathbb{R}^n \), then \( u \) is a linear combination of \( v \) and \( w \).

w. Suppose that \( v_1, v_2, \) and \( v_3 \) are in \( \mathbb{R}^3 \), \( v_2 \) is not a multiple of \( v_1 \), and \( v_3 \) is not a linear combination of \( v_1 \) and \( v_2 \). Then \( \{v_1, v_2, v_3\} \) is linearly independent.

x. A linear transformation is a function.

y. If \( A \) is a \( 6 \times 5 \) matrix, the linear transformation \( x \mapsto Ax \) cannot map \( \mathbb{R}^5 \) onto \( \mathbb{R}^6 \).

z. If \( A \) is an \( m \times n \) matrix with \( m \) pivot columns, then the linear transformation \( x \mapsto Ax \) is a one-to-one mapping.