

MATH 341, SELF QUIZ 1

Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.)

- a. Every matrix is row equivalent to a unique matrix in echelon form.
- b. Any system of n linear equations in n variables has at most n solutions.
- c. If a system of linear equations has two different solutions, it must have infinitely many solutions.
- d. If a system of linear equations has no free variables, then it has a unique solution.
- e. If an augmented matrix $[A \ \mathbf{b}]$ is transformed into $[C \ \mathbf{d}]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution sets.
- f. If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.
- g. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some \mathbf{b} , then the columns of A span \mathbb{R}^m .
- h. If an augmented matrix $[A \ \mathbf{b}]$ can be transformed by elementary row operations into reduced echelon form, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.
- i. If matrices A and B are row equivalent, they have the same reduced echelon form.
- j. The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.
- k. If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^m , then A has m pivot columns.
- l. If an $m \times n$ matrix A has a pivot position in every row, then the equation $A\mathbf{x}$ has a unique solution for each \mathbf{b} in \mathbb{R}^m .
- m. If an $n \times n$ matrix A has n pivot positions, then the reduced echelon form of A is the $n \times n$ identity matrix.
- n. If 3×3 matrices A and B each have three pivot positions, then A can be transformed into B by elementary row operations.
- o. If A is an $m \times n$ matrix, if the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.
- p. If A and B are row equivalent $m \times n$ matrices and if the columns of A span \mathbb{R}^m , then so do the columns of B .
- q. If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in \mathbb{R}^3 is a multiple of one of the other vectors, then S is linearly independent.
- r. If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then \mathbf{u} , \mathbf{v} , and \mathbf{w} are not in \mathbb{R}^2 .
- s. In some cases, it is possible for four vectors to span \mathbb{R}^5 .
- t. If \mathbf{u} and \mathbf{v} are in \mathbb{R}^m , then $-\mathbf{u}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$.
- u. If \mathbf{u} , \mathbf{v} , and \mathbf{w} are nonzero vectors in \mathbb{R}^2 , then \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} .
- v. If \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} in \mathbb{R}^n , then \mathbf{u} is a linear combination of \mathbf{v} and \mathbf{w} .
- w. Suppose that \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 are in \mathbb{R}^5 , \mathbf{v}_2 is not a multiple of \mathbf{v}_1 , and \mathbf{v}_3 is not a linear combination of \mathbf{v}_1 and \mathbf{v}_2 . Then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.
- x. A linear transformation is a function.
- y. If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map \mathbb{R}^5 onto \mathbb{R}^6 .
- z. If A is an $m \times n$ matrix with m pivot columns, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is a one-to-one mapping.