

Mark each statement True or False. Justify each answer. Assume that all matrices here are square.

- a. If  $A$  is a  $2 \times 2$  matrix with a zero determinant, then one column of  $A$  is a multiple of the other.
- b. If two rows of a  $3 \times 3$  matrix  $A$  are the same, then  $\det A = 0$ .
- c. If  $A$  is a  $3 \times 3$  matrix, then  $\det 5A = 5 \det A$ .
- d. If  $A$  and  $B$  are  $n \times n$  matrices, with  $\det A = 2$  and  $\det B = 3$ , then  $\det(A + B) = 5$ .
- e. If  $A$  is  $n \times n$  and  $\det A = 2$ , then  $\det A^3 = 6$ .
- f. If  $B$  is produced by interchanging two rows of  $A$ , then  $\det B = \det A$ .
- g. If  $B$  is produced by multiplying row 3 of  $A$  by 5, then  $\det B = 5 \cdot \det A$ .
- h. If  $B$  is formed by adding to one row of  $A$  a linear combination of the other rows, then  $\det B = \det A$ .
- i.  $\det A^T = -\det A$ .
- j.  $\det(-A) = -\det A$ .
- k.  $\det A^T A \geq 0$ .
- l. Any system of  $n$  linear equations in  $n$  variables can be solved by Cramer's rule.
- m. If  $\mathbf{u}$  and  $\mathbf{v}$  are in  $\mathbb{R}^2$  and  $\det[\mathbf{u} \quad \mathbf{v}] = 10$ , then the area of the triangle in the plane with vertices at  $\mathbf{0}$ ,  $\mathbf{u}$ , and  $\mathbf{v}$  is 10.
- n. If  $A^3 = 0$ , then  $\det A = 0$ .
- o. If  $A$  is invertible, then  $\det A^{-1} = \det A$ .
- p. If  $A$  is invertible, then  $(\det A)(\det A^{-1}) = 1$ .