MATH 341, SELF QUIZ 3

Mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.) In parts (a)–(f), $\mathbf{v}_1, \dots, \mathbf{v}_p$ are vectors in a nonzero finite-dimensional vector space V, and $S = \{\mathbf{v}_1, \dots, \mathbf{v}_p\}$.

a. The set of all linear combinations of $\mathbf{v}_1, \dots, \mathbf{v}_p$ is a vector space.

b. If $\{\mathbf{v}_1, \dots, \mathbf{v}_{p-1}\}$ spans V, then S spans V.

c. If $\{\mathbf{v}_1, \dots, \mathbf{v}_{p-1}\}$ is linearly independent, then so is S.

d. If S is linearly independent, then S is a basis for V.

e. If Span $S = V$, then some subset of S is a basis for V.

f. If $\dim V = p$ and Span $S = V$, then S cannot be linearly dependent.

g. A plane in $\mathbb{R}^3$ is a two-dimensional subspace.

h. The nonpivot columns of a matrix are always linearly dependent.

i. Row operations on a matrix A can change the linear dependence relations among the rows of A.

j. Row operations on a matrix can change the null space.

k. The rank of a matrix equals the number of nonzero rows.

l. If an $m \times n$ matrix A is row equivalent to an echelon matrix U and if U has k nonzero rows, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is $m - k$.

m. If B is obtained from a matrix A by several elementary row operations, then rank $B =$ rank A.

n. The nonzero rows of a matrix A form a basis for Row A.

o. If matrices A and B have the same reduced echelon form, then Row $A =$ Row B.

p. If H is a subspace of $\mathbb{R}^3$, then there is a 3×3 matrix A such that $H =$ Col A.

q. If A is $m \times n$ and rank $A = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

r. If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then rank $A = m$.