SELF QUIZ 3 SOLUTION

a. True. This set is $\text{Span}\{\mathbf{v}_1, \ldots \mathbf{v}_p\}$, and every subspace is itself a vector space.

b. True. Any linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}$ is also a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}, \mathbf{v}_p$ using the zero weight on $\mathbf{v}_p$.

c. False. Counterexample: Take $\mathbf{v}_p = 2\mathbf{v}_1$. Then $\{\mathbf{v}_1, \ldots \mathbf{v}_p\}$ is linearly dependent.

d. False. Counterexample: Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$. Then $\{\mathbf{e}_1, \mathbf{e}_2\}$ is a linearly independent set but is not a basis for $\mathbb{R}^3$.

e. True. See the Spanning Set Theorem (Section 4.3).

f. True. By the Basis Theorem, S is a basis for V because S spans V and has exactly p elements. So S must be linearly independent.

g. False. The plane must pass through the origin to be a subspace.

h. False. Counterexample: $\begin{bmatrix} 2 & 5 & -2 & 0 \\ 0 & 0 & 7 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

i. True. This statement appears before Theorem 13 in Section 4.6.

j. False. Row operations on A do not change the solutions of $A\mathbf{x} = \mathbf{0}$.

k. False. Counterexample: $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$; A has two nonzero rows but the rank of A is 1.

l. False. If U has k nonzero rows, then rank $A = k$ and dimNul $A = n - k$ by the Rank Theorem.

m. True. Row equivalent matrices have the same number of pivot columns.

n. False. The nonzero rows of A span Row A but they may not be linearly independent.

o. True. The nonzero rows of the reduced echelon form E form a basis for the row space of each matrix that is row equivalent to E.

p. True. If H is the zero subspace, let A be the 3×3 zero matrix. If dim $H = 1$, let $\{\mathbf{v}\}$ be a basis for H and set $A = [\mathbf{v} \quad \mathbf{v} \quad \mathbf{v}]$. If dim $H = 2$, let $\{\mathbf{u}, \mathbf{v}\}$ be a basis for H and set $A = [\mathbf{u} \quad \mathbf{v} \quad \mathbf{v}]$, for example. If dim $H = 3$, then $H = \mathbb{R}^3$, so A can be any 3×3 invertible matrix. Or, let $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ be a basis for H and set $A = [\mathbf{u} \quad \mathbf{v} \quad \mathbf{w}]$.

q. False. Counterexample: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. If rank $A = n$ (the number of *columns* in A), then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

r. True. If $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then Col $A = \mathbb{R}^m$ and rank $A = m$. See Theorem 12(a) in Section 1.9.