Math 342, Winter 2012

STUDY GUIDE FOR MIDTERM 1

The midterm exam focuses on the main concepts and topics of Sections 5.1-5.7. There may be a few definitions on the exam. The most important definitions include:

Eigenvalue, eigenvector, eigenspace, diagonalization.

A number of questions will require that you give reasons for your answers. These reasons will often involve a reference to a theorem.

Definitions:

Eigenvalue, eigenvector, eigenspace, diagonalizable matrices.
Similar matrices.
Matrix of a linear transformation T relative to a basis $\mathcal{B}$, $[T]_{\mathcal{B}}$.

Theorems:

Chapter 5: Theorems 1, 2, 4, 5 (the diagonalization theorem), 6, and 8.

Important skills:

Find a change-of-coordinates matrix, use this matrix to find a coordinate vector

Determine if a number (vector) is an eigenvalue (eigenvector) of a matrix

Find the characteristic equation and eigenvalues of a 2×2 matrix. Find the eigenvalues of a triangular matrix, listed according to their multiplicities.

Find a basis for an eigenspace.

If A is diagonalizable, find P and D such that $A = PDP^{-1}$. Show how to compute high powers of a diagonalizable matrix.

Find the $\mathcal{B}$-matrix $[T]_{\mathcal{B}}$ of a linear transformation $T : V \to V$ relative to a basis $\mathcal{B}$ of V.

Verify statements involving similarity of matrices.

Find complex eigenvalues and corresponding eigenvectors.

Find a factorization of a 2×2 matrix with a complex eigenvalue, $A = PCP^{-1}$, where the transformation $x \mapsto Cx$ is a composition of a rotation and possibly a scaling transformation. Determine the angle of the rotation and the scale factor.

Find the solution of a difference equation $\mathbf{x}_{k+1} = A\mathbf{x}_k$ in terms of the eigenvalues and eigenvectors of A, and describe the discrete evolution of the dynamical system. Use eigenvectors to describe the directions of greatest attraction and greatest repulsion. Be able to classify the origin as an attractor, a repellor, or a saddle point. Describe how a change of variable can decouple a system of difference equations.

Same for the differential equation $\mathbf{x}' = A\mathbf{x}$.