MATH 253: HOMEWORK 9.

1. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{6^n}{n!}$.

2. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (2\pi)^{2n+1}}{(2n+1)!}$.

3. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$ in terms of standard elementary functions.

4. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$.

5. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{n \cdot 5^n}$.

6. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n \cdot n!}$.

7. Find the sum of the series $1 - \ln(2) + \dfrac{[\ln(2)]^2}{2!} - \dfrac{[\ln(2)]^3}{3!} + \cdots$.

8. Find the sum of the series $\dfrac{1}{1 \cdot 2} + \dfrac{1}{3 \cdot 2^3} + \dfrac{1}{5 \cdot 2^5} + \dfrac{1}{7 \cdot 2^7} + \cdots$.

9. Find the terms through degree 4 of the general power series solution to the equation $y' = xy + 1$ centered at 0. Then find the terms through degree 4 of the solution which satisfies $y(0) = 1$.

10. Find the terms through degree 5 of the general power series solution to the equation $y'' + xy' + y = 0$ centered at 0. Then find the terms through degree 5 of the solution which satisfies $y(0) = 2$ and $y'(0) = 1$.

11. Find the terms through degree 4 of the general power series solution to the equation $(1 - x)y'' + y = 0$ centered at 0.

12. Find the terms through degree 4 of the power series solution to $xy'' + y' + xy = 0$ centered at $x = 1$, satisfying the initial values $y(1) = 0$ and $y'(1) = 2$.

13. For the differential equation $y'' + xy' + 2y = 0$, find the recurrence relation for the coefficients of the power series solutions centered at 0, and find two independent power series solutions through the terms of degree 3. Be sure to show your reasoning in mathematically and notationally correct steps.

14. For the differential equation $(2 + x^2)y'' - xy' + 4y = 0$, find the recurrence relation for the coefficients of the power series solutions centered at 0.

15. Suppose the function $y = y(x)$ is a solution to the differential equation $y'' + xy' + y = 0$, and satisfies $y(0) = 1$ and $y'(0) = 0$. Find $y''(0)$, $y'''(0)$, and $y^{(4)}(0)$.