Solutions to Self-Quiz 2

a. False. The length of the zero vector is zero.

b. True. By the displayed equation before Example 2 in Section 6.1, with $c = -1$, $\| -\mathbf{x} \| = \| (-1)\mathbf{x} \| = |-1| \| \mathbf{x} \| = \| \mathbf{x} \|$.

c. True. This is the definition of distance.

d. False. This equation would be true if $r\| \mathbf{v} \|$ were replaced by $| r | \| \mathbf{v} \|$.

e. False. Orthogonal *nonzero* vectors are linearly independent.

f. True. If $\mathbf{x} \cdot \mathbf{u} = 0$ and $\mathbf{x} \cdot \mathbf{v} = 0$, then $\mathbf{x} \cdot (\mathbf{u} - \mathbf{v}) = \mathbf{x} \cdot \mathbf{u} - \mathbf{x} \cdot \mathbf{v} = 0$.

g. True. This is the "only if" part of the Pythagorean Theorem in Section 6.1.

h. True. This is the "only if" part of the Pythagorean Theorem in Section 6.1 where **v** is replaced by $-\mathbf{v}$, because $\| -\mathbf{v} \|^2$ is the same as $\| \mathbf{v} \|^2$.

i. False. The orthogonal projection of **y** onto **u** is a scalar multiple of **u**, not **y** (except when **y** itself is already a multiple of **u**).

j. True. The orthogonal projection of any vector **y** onto W is always a vector in W.

k. True. This is a special case of the statement in the box following Example 6 in Section 6.1 (and proved in Exercise 30 of Section 6.1).

l. False. The zero vector is in both W and $W^{\perp}$.

m. True. See Exercise 32 in Section 6.2. If $\mathbf{v}_i \cdot \mathbf{v}_j = 0$, then $(c_i \mathbf{v}_i) \cdot (c_j \mathbf{v}_j) = c_i c_j (\mathbf{v}_i \cdot \mathbf{v}_j) = c_i c_j 0 = 0$.

n. False. This statement is true only for a *square* matrix. See Theorem 10 in Section 6.3.

o. False. An orthogonal matrix is square and has *orthonormal* columns.

p. True. See Exercises 27 and 28 in Section 6.2. If U has orthonormal columns, then $U^T U = I$. If U is also square, then the Invertible Matrix Theorem shows that U is invertible and $U^{-1} = U^T$. In this case, $U^T U = I$, which shows that the columns of U^T are orthonormal; that is, the rows of U are orthonormal.

q. True. By the Orthogonal Decomposition Theorem, the vectors $\text{proj}_W \mathbf{v}$ and $\mathbf{v} - \text{proj}_W \mathbf{v}$ are orthogonal, so the stated equality follows from the Pythagorean Theorem.

r. False. A least-squares solution is a vector $\hat{\mathbf{x}}$ (not $A\hat{\mathbf{x}}$) such that $A\hat{\mathbf{x}}$ is the closest point to **b** in Col A.

s. False. The equation $\hat{\mathbf{x}} = (\mathrm{A}^{\mathrm{T}}\mathrm{A})^{-1}\mathrm{A}^{\mathrm{T}}\mathbf{b}$ describes the *solution* of the normal equations, not the matrix form of the normal equations. Furthermore, this equation makes sense only when $A^T A$ is invertible.