Math 316 (Fund. of analysis), Winter 2018
Quiz 1

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Name:

Q1		6
Q2		6
Q3		8
Q4		20
Q5		10
Total		50

General notes:

1. Terminology:
   - Supremum = least upper bound.
   - Injective = 1-to-1 = into.
   - Surjective = onto.
2. Justification: On an exam, by default, you should justify your statements. If an example or counterexample is the best justification, you should provide one. Otherwise, you should provide a brief conceptual explanation (**one or two sentences**).
   - To justify that "Every closed interval contains 0" is false, you should provide a counterexample, such as $[1, 2]$.
   - To justify that "Some closed interval contains 0" is true, you should provide an example, such as $[-1, 1]$.
   - To justify that "Every closed interval contains some nonzero number" is true, you should explain. "For any interval $[a, b]$, either $a \neq 0$ or $b \neq 0$."
3. If I want a complete proof, I will say "Prove that ..."
4. If I don't need any justification, I will say "no justification necessary." No justification is needed for a definition.

1. **(6 pts)** Let A be a subset of $\mathbb{R}$. Give a formal definition of the *infimum* or *greatest lower bound* of A.

2. **(6 pts)** Consider the set

$$B = \{3 - \frac{1}{n} \mid n \in \mathbb{N}\} \cup (497, 500)$$

inside $\mathbb{R}$. Does B have more than one supremum? Compute every supremum of B (no justification necessary).

3. **(8 pts)** Suppose C and D are nonempty subsets of $\mathbb{R}$ which are bounded above, but not bounded below. Define the set

$$C \cdot D = \{c \cdot d \mid c \in C, d \in D\}.$$

Does $\sup(C \cdot D)$ exist? If so, find a formula for it. If not, find a counterexample.

4. **(20 pts)** For each of the following statements, is it true or false? Justification is required.

   (a) Any open interval in $\mathbb{R}$ contains an irrational number.

   (b) Every nonempty subset of $\mathbb{R}$ has a supremum.

   (c) There is no collection of open intervals I_n such that $\bigcap_{n\in\mathbb{N}} I_n$ is a closed interval.

   (d) The set $\{\text{even numbers}\} \cup \{\text{multiples of } 7\}$ is countable.

5. **(10 pts)** Let $a < b$ be real numbers, and consider the set $T = \mathbb{Q} \cap [a, b]$. Prove that $\sup T = b$.