WARTHOG 2016
Exercises for Thursday morning

1. Describe $\mathrm{Hilb}^n(X)$ explicitly when X is the germ of the singularity given by the equation $x^2 = y^5$. Find the generating series of Poincare polynomials and see that it agrees with the bottom row of $\overline{HHH}(T(2,5))$. Can you do the same thing for $T(3,4)$?

2. Let
$$X(n,m) \subset \mathrm{Hilb}^n(X) \times \mathrm{Hilb}^{n+m}(X) = \{(I,J) \,|\, M \cdot I \subset J \subset I\},$$
and let $\pi : X(n,m) \to \mathrm{Hilb}^n(X)$ be the natural projection. Show that if $I \in \mathrm{Hilb}^n_r(X)$ (so its minimal number of generators is r), then $\pi^{-1}(I) = \mathrm{Gr}(m,r)$ is the Grassmanian of m-planes in $\mathbb{C}^r$. Deduce that the generating series
$$\sum_{n,r} q^{2n} \prod_{i=1}^{r} (1 + t^{2i-1}a^2) \mathcal{P}(\mathrm{Hilb}^n_r(X) = \sum_{n,m} q^{2n} t^{m^2} a^{2m} \mathcal{P}(X(n,m))$$

3. Let X be the germ of the singularity given by the equation $x^n = y^m$. Show that any ideal in $\mathcal{O}_X$ is generated by at most $\min(n,m)$ generators.

4. Let $S_{n,m} \subset R$ be the semigroup generated by n and m. Show that
$$\sum_{s \in S_{n,m}} q^{2s} = q^{\mu-1} \overline{P}(T(n,m))|_{a=1}.$$
Can you generalize this statement to more general singularities?