For the week and a half we are going to prove some of the patterns you’ve noticed in factoring \( x^n - 1 \). Then in the last week and a half I want to cover the material in section 4.3 in the book. Hopefully both these topics will help you get a better understanding of the fundamental ideas we have been discussing in fairly concrete situations.

You should REREAD sections 4.1 and 4.2 in the book. You should have read these before already – but now is a good moment to look back at them – it should start to make some sense this time!

- Exercises 4.1: 6, 18(a)(b)(c)
- Exercises 4.2: 3(e)
- Exercises 3.3: 3(b)(d), 8, 10.

Compute the cyclotomic polynomials \( \Phi_n(x) \) for \( n = 1, 2, \ldots, 16 \) (you should be able to read the first 12 off from the table of factors of \( (x^n - 1) \) you created last week – remember \( \Phi_n(x) \) is one of its irreducible factors of degree \( \phi(n) \).) Question: for which \( n \) are ALL the coefficients of the polynomial \( \Phi_n(x) \) positive? Do you see any pattern? Probably not, since as far as I am aware it is an unsolved problem to characterize the positive integers \( n \) for which all coefficients of the polynomials \( \Phi_n(x) \) are positive.