Legalize, Tax, and Deter: Optimal Enforcement Policies for Corruptible Officials

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Abstract

Corruption of law enforcers remains a significant problem in many countries. We propose a model where deterrence of a harmful activity (such as trade in illicit drugs) is hindered by corruption, but standard anti-corruption policies might be too costly to implement. We show that an alternative tax-and-legalize policy can yield significant benefits, especially in poor countries with weak institutions and for activities that are not too harmful. Using a tax-and-legalize scheme, the government reduces the cost of implementing standard anti-corruption policies, easing the fight against corruption. However, a tax-and-legalize policy could increase the equilibrium number of harmful activities.

JEL Classification: D73, K42, O10, H21

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1 Introduction

The corruptibility of law enforcers remains a significant concern in many developing and advanced economies. Among the possible anti-corruption policies, the economic literature has mainly focused on compensation schemes that penalize corruption and reward honest behaviors (Mookherjee and Png [1995], Mishra [2002], and Bose [2004]). While these policies

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have the potential to effectively limit corruption, their implementation has proven to be unsatisfactory in many countries (Transparency International [2010]). What could explain these failures? A reason may be that stamping out corruption is costly, and in some countries tolerating it might be socially optimal (Besley and McClaren [1993], Acemoglu and Verdier [2000], and Baç, and Bag [2006]).

The primary objective of this paper is to show that legalization is useful in cases where traditional anti-corruption policies are costly to implement. In a setting where individuals engage in harmful activities and bribe officers to avoid being sanctioned, we study the effect of implementing a tax-and-legalize scheme. This scheme allows individuals to freely undertake the sanctioned activity in exchange for the payment of a tax or reduced fine. We will show that legalization changes the enforcement problem, turning it from one of apprehension of sanctioned acts, to one of detection of tax evasion. However, the potential problem of corruption remains: just as before, tax evaders can bribe audit officers. Yet, this paper will demonstrate that legalization in some instances reduces the cost of implementing an effective (and otherwise expensive) anti-corruption scheme, thus solving the corruption problem.

To show this point, we build a simple model of enforcement in which individuals generate a social harm when engaging in a certain activity (such as pollution or dealing with illegal drugs) that provides them with a private gain. The government hires officers to monitor the population and fine miscreants, and can decide whether to allow bribery to take place or, through appropriate incentives, ensure that officers never accept bribes. We model the enforcement problem such that bribes lower deterrence but are otherwise efficient: they generate a transfer from miscreant to officers that avoids socially costly bureaucracy and red tape. This setup allows our model to obtain many expected predictions that conform with the existing literature. For instance, we find that poorer countries are more likely to adopt corrupt enforcement, because anti-corruption policies are more expensive at lower income levels. We also find that countries that are more tolerant of corruption or with high red tape levels are more likely to adopt corrupt enforcement regimes. More surprisingly, we find that
the corruptibility of officers imposes significant welfare costs precisely in countries with low levels of wealth, where corruption is most likely to occur. At higher levels of wealth, on the other hand, the corruptibility of agents matters less and less for welfare: the tools available to the government are sufficiently flexible that corruption avoidance is inexpensive.

Given this model, we analyze the effect of a tax-and-legalize policy. Individuals have the option to freely engage in the harmful activity, then pay a tax to the government. Since tax avoidance is an option, tax evaders would still face sanctions once audited, and therefore can still attempt to bribe officers. Thus, the corruption problem does not go away with a tax-and-legalize scheme. However, those individuals who do pay the tax have acted legitimately, and therefore need not bribe the tax enforcers. Whenever one individual pay the tax, there is one less anti-corruption incentive that needs to be paid out. This makes anti-corruption policies sufficiently cheap that, as it will be shown, are completely and comprehensively adopted in a legalized regime. This is particularly useful when such policies are otherwise costly, which occurs at low levels of wealth in the model, precisely where corruption is the preferred policy. Our welfare predictions are therefore quite stark: poor countries where an illegal activity begets corruption should legalize, regardless of how tolerant of corruption that country is.

The use of a tax-and-legalize scheme does lead to a potentially unpleasant effect: in general, we find that more individuals engage in the socially harmful activity when it is legal, even when the activity is appropriately taxed. Unlike most prior (mostly empirical) literature, this is not due to changes in the demand function of consumers or motivated by the introduction of a revenue concern to the government: our model keeps both demand function and government revenue concern constant. Instead, the reason for the increase in the harmful activity is that, absent legalization, there is too much enforcement (something that was also previously noted by Mookherjee and Png [1995]). Correcting this over-enforcement is a property of legalization that, in a political economy context, could be considered controversial.

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1 Empirically, legalization has been found to increase the quantity demanded of the previously illegal good (Saffer, and Chaloupka, (1999), DiNardo and Lemieux (2001), and Anderson and Rees (2011)).
while society is better off with higher levels of the harmful act and a lower government budget, individual voters who easily observe the first and not the latter might reject legalization. Over-enforcement is less likely to occur at very low levels of wealth and for activities that are not too harmful, suggesting that legalization could be more appealing in developing countries and for lesser illegal activities.

The theory presented here is empirically relevant, particularly (but not only) for the regulation of illicit drugs. Many poor countries have already legalized narcotics that are illegal elsewhere. For instance, chat, a narcotic plant, is freely traded, taxed, and consumed in Ethiopia, Yemen and Somalia. It remains illegal elsewhere. Bolivia has legalized the sale and consumption of coca leaves, although more processed and harmful forms (a category that includes cocaine) remain illegal. The call for broader legalization of other illegal drugs is being increasingly made in policy circles by prominent figures concerned with corruption. In 2010 former president of Mexico Vincente Fox argued in favor of drug legalization, in part on corruption grounds. The presidents of both Bolivia and Uruguay are actively pursuing more lax legalization policies, against the wishes of many western countries.

There are other instances of legalization or self reporting policies in areas where the possibility of corruption is great. As part of a larger anti-corruption push, off-track betting was legalized in Hong Kong in the seventies; the legalization eliminated an important source of bribes for the police force, and police corruption was eliminated shortly after (Klitgaard [1988]). While big game hunting is generally illegal in Africa, countries such as Malawi, Tanzania and Zambia have adopted hunting permits as a form of legalization for a crime –

\footnote{The consumption and trade of cannabis has been legalized in many Western countries and US states too; such legalization is generally driven by considerations not directly considered here, such as incarceration policies. Nonetheless, our model can explain this evidence. Our results suggest that legalization can be optimal even in countries where effective anti-corruption policies are in place, especially for activities that are not too harmful.}

\footnote{Among the various voices in the debate over legalization of things other than drugs, it is worth mentioning Basu (2011) who recently suggested legalizing bribe giving, while keeping sanctions on bribe taking. He observed that, when the entire punishment from corruption is concentrated on the bribe taker, the bribe giver has an incentive to report the bribe exchange. Anticipating this, the bribe taker will be much less inclined to take the bribe in the first place. While we model legalizing the underlying activity and not the act of bribing, both models hinge on providing an alternative outside option to the bribe giver.}
poaching – that is difficult to control.⁴

More broadly, the scheme we present here extends beyond legalization, and fits into a general class of models where corruption is reduced by empowering individuals to bypass officials (Motta [2012], and Burlando and Motta [2013]). Such schemes are becoming increasingly prevalent. In 2005, India introduced a Freedom of Information Act (the RTIA), which allows citizens to inquire about any governmental activity—including the status of a person’s own permit applications. Peisakhin (2012) and Peisakhin and Pinto (2010) showed in randomized trials that RTIA requests provide an alternative way to petition for a permit, which ultimately reduces bribe payments. Private companies in developing countries also use such schemes to get around their own corruptible employees. In India, the tatkal system was introduced by the phone company to provide people with an alternative to paying bribes for quick phone line installations. The Bharat Sanchar Nigam company guaranteed speedy connections to those willing to sign up and pay into the program. The same tatkal system was later adopted by the railways as a way for customers to avoid long waiting lists or paying bribes to intermediaries for a quick ticket. In many instances, self reporting can become operational following a technological innovation. In Tanzania, customers could consume electricity for years without ever paying their bills. This was quite common until TANESCO introduced ‘smart’ electronic meters which require payment before supplying electricity, bypassing the corrupt workers. This system was borrowed from the cell phone industry and has been successfully adopted in the water, telecommunications and trash disposal sectors.⁵

The rest of the paper is organized as follows: after a discussion of the current theoretical literature, section 2 discusses the model of enforcement sans corruption; section 3 studies the effects of corruption; section 4 introduces the tax and legalize scheme. Section 5 concludes.

⁴That some legalization of wildlife hunting and trade can be helpful and beneficial for conservation is a concept that is not lost for several environmental groups. In their 2008 joint report, TRAFFIC and the WWF state that “Policies that criminalize the wild meat trade have not been effective to bringing it under control...greater consideration of alternative management scenarios, including legalizing hunting and trade of certain wild species for meat is therefore required” (Roe, 2008).

⁵Top-up systems or pre-paid services are different from self reporting in one important aspect: the client has no choice but to pre pay if he wants the service. In law enforcement, miscreants always have a choice not to pay.
Relation to the literature  

There is a growing literature on the economics of corruption and legalization in law enforcement, which is often referred to as self reporting. Under self reporting, the harmful activity is illegal, and remains so. Kaplow and Shavell (1994) introduced the idea of self reporting in law enforcement\(^6\) in their model, self reporting saves money by reducing the number of officers needed to monitor the population\(^7\) These cost savings hinge on the enforcement agency being able to separate self reporters from the rest of the population, which is not generalizable to many contexts where legalization would be useful. Polinsky and Shavell (2001) consider a setup similar to ours but do not explicitly model the source of the deadweight losses from corruption, and do not explore the role of self reporting in corruption\(^8\).

The model we present here can be thought of as a self reporting model, because individuals carry out the harmful acts first, and only later declare the acts through the payment of the tax. An alternative legalization scheme would have an individual having to purchase a permit from a bureaucrat before engaging in the sanctioned activity. Technically, our results would hold irrespective of the timing. However, in the context of the permit, the corruption problem could arise anew: a bureaucrat could decide to hold up the issuing of the permit in exchange of a bribe (see Bardhan [1997] for a thoughtful discussion). This problem is avoided by our scheme because the acts can be reported after they have been committed. In practice, we believe that this distinction is becoming less important: creative use of freedom of information acts, as well as new technologies that allow for the remote application of taxes, levies, and permit requests, help avoiding bureaucratic holdup. We also sidestep another important distinction between legalization and self reporting, namely the psychological effect of making an illegal activity legal (what Miron and Zwiebel [1995] refer

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\(^7\) The literature on self reporting highlights other relevant aspects of the policy in environments without corruption. See Innes (1999, 2000, 2001). For leniency programs see Motta and Polo (2003) and Buccicossi and Spagnolo (2005).

\(^8\) See also Rose-Ackerman (1994), Hindriks, Keen, and Muthoo (1999), Livernois and McKenna (1999), and Malik (1990).
to the “respect for the law” effect.) In light of this effect, the choice of how to frame the policy might go well beyond a simple labelling or semantic issue.

Perhaps closest to our paper is Andrianova and Melissas (2008). Using a different setup, they study when governments choose to legalize an activity when officers are corrupt. They assume that once the activity is legalized, the government imposes no reduced fines or taxes, and employs no enforcement. While this might be useful in some contexts—say, when choosing zero tariffs to reduce corruption among import inspectors—their model cannot explain the role of legalization in context where harm control remains important.

2 The model

2.1 Structure

There is a measure 1 of risk-neutral citizens who cause a harm to society of \( h > 0 \) by committing an act that provides a private gain of \( x \). The gain is distributed with a continuous density function \( g(.) \), which is everywhere strictly positive over the support \([0, \infty)\). We assume a continuous cumulative distribution function \( G(.) \) and, for simplicity, a constant hazard rate function \( \frac{g(x)}{1-G(x)} = \lambda \). Absent enforcement problems, a welfare maximizing government would allow only those citizens whose private benefit is \( x \geq h \) to carry out the act. However, the act is not directly observable, and the government must engage in costly monitoring and enforcement. To this purpose, the government employs a measure \( p \) of officers. They are responsible for monitoring the population and reporting violators to a court of law, which in turn imposes fines. Enforcement works as follows. Each officer is randomly matched with a citizen, and the officer immediately learns whether the citizen has carried out the sanctioned activity. The officer's information is hard and verifiable in a court of law. The officer can report the violation to the judiciary system, which levies a fine \( f \) to the offender. The maximum level of sanction is \( \bar{f} \), which may be interpreted as an individual’s wealth.

\(^9\)All results in the paper are generalizable to a nondecreasing hazard rate function.
2.1.1 Corruption

A corruptible officer will consider accepting a bribe $b$ from the citizen in exchange for her silence. The size of the bribe is reached through a bargaining process. We assume that, while miscreants are willing to pay any bribe that is less than the fine, i.e. $b \leq f$, the bribe cannot be as large as the fine:

**Assumption (Corruption tolerance):** the maximum agreeable bribe cannot be larger than a fraction $\sigma \in (0, 1)$ of the fine.

In our model, the assumption introduces a limit to deterrence in corrupt regimes in a straightforward way. It corresponds to the reality of many bribery situations, where the bribe giver settles for a bribe that is much smaller than the full sanction. Theoretically, there are several ways to explain this condition. At its simplest, the assumption states that the larger the bribe payment demanded by an officer, the greater is the chance of detection (as in Emerson, [2006]). Bribes that are too large become either detectable to the press and scandalous to society, while a relatively small side payment would be either undetected or acceptable. Thus, our $\sigma$ measures society’s tolerance for corruption. Societies that are intolerant (low $\sigma$) find bribery unacceptable, and only small exchanges (that can perhaps be confused as acts of kindness rather than corruption) are possible. On the other hand, countries that are tolerant (high $\sigma$) have societies where gift exchanges (which can mask bribery) are common or consider bribe payments as a way of life. In either case, a collusive agreement where bribes are too high would be uncovered by the public as blatant corruption, the officer would lose the ill-gotten wealth, and the offender would be made to pay the full fine $f$.

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10 The prevalence of gifts exchange traditions allows individuals to mask corruption (Klitgaard, 1988), and some societies may just be more accepting when someone offers bribes to public officials (Lambsdorff, 2007).

11 There are other possible ways to explain our assumption. Even if miscreants’ ability to pay is greater or equal to the fine $f$, they might be unable to transfer more than a fraction $\sigma f$ of this wealth in bribes. This could come from the fact that the fraction $(1-\sigma)f$ is held in assets that are not easily transferrable, or whose ownership is hard to verify at the time of the bribe transfer, such as boats, landholdings, or residential units located in another location or country. While a corrupt officer might be unable to take possession of these assets, a government can reasonably be expected to find, confiscate, and sell these items. Second, the assumption is consistent with a model where part of the punishment is non-monetary in nature, wherein only a fraction $\sigma$ of the punishment is extracted in monetary form and the rest administered via revenue-neutral imprisonment. Finally, the assumption would arise in a model where bribe exchanges suffer transaction
As an anti-corruption measure, the government can offer bonuses or incentives $i$ that are paid when an officer reports an offender. Because acceptance of the bribe implies foregoing the incentive, the officer will consider only bribes that have the following characteristic: $i \leq b \leq \sigma f$. The agreement on the bribe is reached through a bargaining process, with weights to the officer of $\mu \in (0, 1)$. The equilibrium bribe is given by the following expression:

$$b(i, f) = \min \{(1 - \mu)i + \mu f, \sigma f\},$$

(1)

that is, the bribe level is fully characterized by the incentive and the fine. Note that, regardless of how the bargain power is distributed between the two parties, corruption is eliminated whenever the incentive level $i$ meets or exceeds the following no-collusion condition:

$$i \geq \sigma f \equiv i^{nc}.$$  

(IC)

When $i < i^{nc}$, a bribe will be agreed upon, violations are not reported, and the government neither pays incentives, nor receives any income from the fine. When $i \geq i^{nc}$, no bribes are exchanged, and therefore all violations are reported, all fines are levied, and all incentives are paid.

### 2.1.2 Enforcement costs

Officers are risk neutral with reservation utility $v$, where $v$ is strictly positive and finite. The government must offer a salary package that, in expectation, leaves officers with at least their reservation utility. Namely, the base wage $w$ and incentive $i$ must satisfy the following participation constraint:

$$w + \Pr(\text{audit success}) \max[b(i, f), i] \geq v$$

(PC)

costs, similar to those existing in the literature of corruption (Tirole, 1992).
where \( \max[b(i, f), i] \) represents the gains obtained from either the incentive or the bribe. In addition, we assume that officers are shielded by some degree of limited liability.

**Assumption (Limited liability):** Officers’ wages cannot fall below a certain threshold:

\[
w \geq \bar{w} \equiv 0. \tag{LL}
\]

Limited liability is one way to generate a “cost of corruption”; see Burlando and Motta (2012) for an alternative modeling strategy that relies on risk aversion and that generates very similar insights. The assumption is particularly important in the context of many developing countries, where bureaucrats and law enforcement officers are generally very poorly paid, and often go unpaid. A less common occurrence is that they receive *negative* wages, although there are documented circumstances (see Besley and McClaren [1993]) in which enforcement officers essentially pay superiors for the right to enforce (and collect bribes). Even in those cases, it is likely that there is a limited liability constraint, with the minimum wage somewhere below zero. Fortunately, our paper can be easily generalized to those situations. We assume \( \bar{w} = 0 \) for the sake of exposition\(^\text{12}\).

### 2.1.3 The government

We conclude the model setup by introducing a benevolent, welfare maximizing government whose contribution of enforcement expenditures \( B \) on the larger total government budget consist of wage payments net of revenues from fines. The budget process has two distortions. First, the budget is raised through distortionary taxation from the citizenship. A budget \( B \) costs society \((1 + d)B\), where \( d \) measures the size of the dead weight loss and is normalized to \( d = 1 \).\(^\text{13}\) Second, the revenue generating process is subject to some bureaucracy and red

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\(^{12}\)Implicitly, a second limited liability assumption states that \( w + \max(b, i) \geq 0 \). In equilibrium, there will never be a case where bribes or incentives are negative, so the condition is never binding.

\(^{13}\)This is a critical distinction between our approach of corruptible enforcement and the canonical approaches of enforcement in Kaplow and Shavell (1994) or Polinsky and Shavell (2001), where the government budget does not include the revenues from fines collected and the effect of distortionary taxation. From this perspective, our framework is similar in nature to setups more commonly found in the literature of corruption, such as in Besley and McClaren (1993) and Laffont and Tirole (1993).
tape, such that only a fraction $\alpha \leq 1$ of revenues are accounted for and can be utilized. The remaining portion of fine income is destroyed by red tape. We understand $\alpha$ as a measure of efficiency in a country’s government.\footnote{A modified version of costly imposition of fines is found in Polinsky and Shavell (2000).}

The social welfare function depends on the type of enforcement regime (clean or corrupt) and on the number of individuals who commit the sanctioned act. We denote by $\hat{x}$ the expected sanction faced under a certain enforcement regime. $\hat{x}$ is also a handy measure of effective enforcement and social harm. Welfare is

$$W(\hat{x}; i, f, w, p) = \int_{\hat{x}}^{\infty} (x - h) g(x) dx - B(\hat{x}; i, f, w, p),$$

where the budget function $B(\hat{x})$ will be further specialized in the next section. Note that the enforcement regime is characterized by the optimal selection of four policies: the number of inspectors, $p$; the base wage, $w$; the incentive pay, $i$; and the fine to miscreants, $f$.

**Further assumptions and timing of the model** The timing of the model works as follows. First, the government announces the full set of policies. Agents observe these policies and decide whether to engage in the harmful act. Then, officers randomly pick agents for monitoring, discover whether there was harm, and either assess the fine or agree on the bribe.

We make three additional assumptions on the parameter space, whose purpose is to keep the analysis pertinent to probabilistic enforcement by avoiding possible corner solutions where there is either perfect enforcement or no enforcement at all.

**Assumption A1:** $\bar{f} > \frac{h}{1 + \alpha} - \frac{\sigma - \alpha}{\lambda(1 + \alpha)} > 0$

**Assumption A2:** $\bar{f} > v/\sigma$

**Assumption A3:** $\bar{f} > h/\sigma$

The assumptions essentially state that the maximum fines that a government can impose are sufficiently large to be worthwhile for deterrence. Implicitly, they also state that the
social harm $h$ is sufficiently large, relative to parameters. Our main results extend to the case where corner solutions are considered, but the presentation would suffer and become much more involved.

2.2 Benchmark: no corruption

We start with an economy where enforcement agents cannot collude and are always honest. It follows directly that incentives need not be offered, and wage rates remain always at the lowest level possible: $w = v$. Moreover, given that officers never accept bribes, a citizen that commits the illegal act faces an expected sanction $\hat{x} = pf$. The total quantity of crime is $1 - G(pf)$, which is also the probability of a successful audit. In total, thus, the government can expect to receive $\alpha pf[1 - G(pf)]$ in revenues from fines. With this characterization, it is straightforward to derive the welfare function. This will take into account the level of criminality, the wage bill, and revenues from fines:

$$W_{\text{honest}}(p, f) = \int_{pf}^{\infty} (x - h)g(x)dx - pv + \alpha pf[1 - G(pf)].$$

(3)

It is readily observable that the optimal enforcement policy requires maximal fines $f = \bar{f}$. If fines are not maximal, the government can increase welfare by increasing the fines and reducing the amount of officers such that the enforcement level $\hat{x} = pf$ remains constant. The first and last term in equation (3) are a function of the enforcement level, and remain unchanged. However, the term in the middle is the wage bill, which is thus reduced. We lastly characterize the optimal size of the enforcement force, $p^*$. To determine $p^*$, differentiate (3) with respect to $p$ using $f = \bar{f}$ to obtain

$$\frac{\partial W_{\text{honest}}}{\partial p} = \bar{f}(h - p\bar{f})g(p\bar{f}) - v + \alpha f[1 - G(p\bar{f})] - \alpha p\bar{f}g(pf)\bar{f}$$

This expression will be negative for $p = 0$ if $v$ is large enough, so $p^* = 0$ is possible. Moreover, the expression can be positive for $p = 1$, so $p^* = 1$ is also possible. To keep the model as
simple as possible, we always focus only on interior solutions: that is, we limit the parameter space so that the government wants *some* enforcement, and ignore those instances where it is optimal not to enforce, or to enforce every single case. Assumptions 1-3 guarantee this. An interior solution must satisfy the first-order condition that \(\frac{\partial W_{\text{honest}}}{\partial p} = 0\). Making use of the fact that the hazard rate \(\frac{p(x)}{1 - G(x)} = \lambda\) is constant, the optimal enforcement force \(p^*\) solves

\[
p^* \tilde{f} [1 + \alpha] = h - \left( \frac{v}{fg(p^* \tilde{f})} - \frac{\alpha}{\lambda} \right)
\]

The left side is the net social loss from deterring the marginal individual, who would have obtained \(p^* \tilde{f}\) had he committed the act and society would have obtained expected revenue from fines \(p^* \tilde{f} \alpha\). The left side is the net social gain from deterring the same individual, the harm avoided minus the net enforcement cost of deterring him.

While we cannot characterize the solution to this equation in a closed-form solution, it is evident that, for any given set of parameter values, the left hand side of the equation is strictly decreasing in \(p\) while the right hand side is strictly increasing in \(p\); the solution exists and is unique.

In addition, \(p_h^* > 0\), \(p_\alpha^* > 0\) and \(p_v^* < 0\). The first derivative simply states that actions that have larger negative spillovers receive more enforcement. The second derivative implies that administrations that are more efficient at collecting fines (have higher \(\alpha\)) have also a higher enforcement level. The implication is that enforcement is affected by a revenue concern. The final derivative shows that enforcement decreases with \(v\), which measures the cost of an agent.

### 3 Enforcement in the presence of corruption

We next consider the case where corruption is possible. In solving for the optimal enforcement policy, it is useful to consider separately the case where incentives are sufficiently high so that officers do not accept bribes, and the case where incentives are low and officers are corrupt. We refer to these two situations as *corrupt* and *clean*, and we index these two regimes by
\[ j = c, nc \] respectively.

### 3.1 Clean regime

To begin, suppose that the incentive provided is \( i \geq i^{nc} \). As above, officers do not accept bribes, so that enforcement is \( \hat{x} = pf \) and revenues from fines are \( \alpha pf[1 - G(pf)] \). On the cost side, the government pays a base wage bill \( pw \) and an incentive pay \( i \) for each fine that is levied. With these elements, the welfare function is given by

\[
W_{nc}(p, w, i, f) = \int_{pf}^{\infty} (x - h)g(x)dx - pw + p(\alpha f - i)[1 - G(pf)]. \tag{5}
\]

The government maximises this welfare function subject to the incentive constraint \([\text{IC}]\), the limited liability constraint \([\text{LL}]\) and to the participation constraint \([\text{PC}]\) which in this case takes the form

\[
w + i[1 - G(pf)] \geq v \tag{PC_{nc}}
\]

Note that at least one or both participation and limited liability constraints must bind in equilibrium. (When they do not, the base wage \( w \) can be reduced without affecting the enforcement level, and welfare increases.) Which constraint will bind depends on the parameters of the model. Regardless of which constraint binds at the optimum, the following remark applies:

**Remark 1** The optimal policy in the clean regime includes \( i = i^{nc} \) and \( f = \bar{f} \). Enforcement is given by \( \hat{x}_{nc} = pf \).

We describe the general argument for this remark here, leaving the full proof to be part of the optimal policy in proposition 2. Incentives are generally costly to the government whenever \( [\text{PC}_{nc}] \) is slack, and revenue-neutral otherwise. Given that increasing the incentive above \( i^{nc} \) does not serve any purpose in terms of promoting honesty, setting \( i = i^{nc} \) is always an optimal response. A similar argument holds for the fine \( f \). For any fine amount that is not maximal, it can be shown that the government can achieve the same enforcement \( \hat{x} = pf \) by
increasing \( f \) and reducing \( p \), such that enforcement costs are (weakly) lower. Hence, setting the maximal fine \( f = \bar{f} \) is also optimal.

The clean regime \( \text{(5)} \) is similar to the benchmark, honest case. In fact, it is readily observed that the two are identical and lead to the same welfare whenever \( \text{(LL)} \) is slack. Once \( \text{(LL)} \) binds, benchmark welfare is unattainable and corruption is socially costly. We can measure the cost of corruption \( C_{nc}(\hat{x}) \) to be the difference in welfare between the clean and the benchmark case, for any given enforcement level \( \hat{x} \), using \( f = \bar{f} \) and \( i = i_{nc} \). Formally, it is

\[
C_{nc}(\hat{x}) \equiv W_{\text{honest}}(\hat{x}) - W_{nc}(\hat{x}) = \begin{cases} 0 & \text{for } v \geq \tilde{v}_\sigma \\ p \{ \sigma \bar{f}[1 - G(\hat{x})] - v \} & \text{for } v < \tilde{v}_\sigma, \end{cases}
\]

where \( \tilde{v}_\sigma \equiv \sigma \bar{f}[1 - G(p \bar{f})] \) is the value of market wages where \( \text{(LL)} \) binds exactly. This threshold value is increasing in \( \sigma \). In the top panel of figure 1 we simulate the model and plot \( C_{nc}(\hat{x}) \) against market wages \( v \), for an arbitrary enforcement level, and for two values of \( \sigma \). Details on the simulation and the parameters used are in Appendix C. The cost of corruption is zero for \( v > \tilde{v}_\sigma \). When countries are sufficiently wealthy or have sufficiently strong institutions, the corruptibility of officers does not create distortions because the cost of paying high incentives is entirely neutralized by lowering the base salary \( w \). When market wages are below \( \tilde{v}_\sigma \), the government cannot neutralize the high incentives costs, owing to limited liability. The limited liability threshold is reached either because wages \( v \) are low, or because of high tolerance for corruption \( \sigma \), which increases the equilibrium bribe and raises the incentive \( i_{nc} \) required to keep the officers clean. “Clean” enforcement becomes socially costly, with the cost increasing the poorer the country or the more tolerant to corruption the institutions are. Under plausible parameter values, our simulations suggest that the cost

\[15\] A comparison between the clean and the benchmark regime evaluated at their respective optimal \( p \) is discussed in the next session, when legalization is introduced. Comparing the regimes at their optimal policies does not allow us to account for the direct cost of corruption because the level of enforcement in the two regimes is allowed to vary.
of corruption can be substantial for sufficiently poor countries.

**Optimal “clean” enforcement**  We are now left with characterizing the optimal interior \( \hat{x}_{nc} \) in the clean regime. As it will be clear later in the discussion, Assumptions 1-3 prevent corner solutions, while relaxing these assumptions would not change our results. For the sake of exposition, define two market-wage threshold values \( \bar{v}_{\sigma} \equiv \sigma \bar{f} [1 - G(h(1+\alpha-\sigma))^{\sigma-1}/\lambda(1+\alpha-\sigma))] \) and \( \tilde{v}_{\sigma} \equiv \sigma \bar{f} [1 - G(h(1+\alpha-\sigma))^{\sigma-1}/\lambda(1+\alpha-\sigma))] \). The following proposition characterizes the optimal policy.

**Proposition 2** The optimal interior enforcement policy with high-powered incentives includes fines and incentives as defined in remark 4 and enforcement \( \hat{x}_{nc} \) as follows:

a. When \( \bar{v} > \bar{v}_{\sigma} \), only \( (PC_{nc}) \) binds, the enforcement problem is equivalent to the benchmark "honest" case equation (3), and \( \hat{x}_{nc} \) is given by first order condition (4).

b. When \( \bar{v} \leq \bar{v}_{\sigma} \), both \( (LL) \) and \( (PC_{nc}) \) bind, and the enforcement level is \( \hat{x}_{nc} = G^{-1}(1 - \bar{v}/\bar{f}) \).

c. When \( \bar{v} < \bar{v}_{\sigma} \), \( (LL) \) binds, \( (PC_{nc}) \) is slack, and the enforcement level is \( \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \).

**Proof.** See appendix.  ■

When \( v > \bar{v}_{\sigma} \) (case a.), the incentives paid out are perfectly compensated by a reduction in the base salary (i.e., the participation constraint binds), leading to zero expected rents to officers, no welfare losses from corruption, and benchmark welfare. In case b., countries with intermediate wealth \( (v \in (\bar{v}_{\sigma}, \bar{v}_{\sigma})) \) optimally decrease the base wage \( w \) to the minimum level allowed by limited liability. However, officers still earn zero rent: in expected value, they obtain exactly their market wage \( v \). The government achieves this at the cost of distorting enforcement away from the benchmark level. In fact, the distortion is such that there is *over-enforcement* with respect to the benchmark. Over-enforcement reduces the probability that an officer earns rents, and causes a reduction both in harmful activity, and in the number
of times incentives are paid. This relaxes \( \text{PC}_{nc} \) and thus prevents the officers from earning positive corruption rents.

Avoidance of rents via over-enforcement becomes more and more costly the poorer the country. When \( v \leq \tilde{\nu}^{nc}_\sigma \) (case c.), the distortion necessary is sufficiently large that it becomes optimal to pay out rents—\( \text{PC}_{nc} \) becomes slack. Below that threshold, optimal enforcement ceases to depend on \( v \).

Three additional comments are in order. First, in all cases enforcement is (weakly) increasing in \( h \), as expected: ceteribus paribus, the optimal policy involves more enforcement the more severe the act. Second, the more tolerant of corruption a country is, the higher the level of wealth \( \tilde{\nu}^{nc}_\sigma \) and \( \tilde{\nu}^{nc}_\sigma \) where cleaning up corruption becomes socially costly. On a more technical note, the optimal policy allows for maximal fines and binding (IC) but welfare maximization does not require this feature. Alternative policies where \( f < \bar{f} \) and \( i > \sigma f \) are possible, and fully spelled out in the appendix.

### 3.2 Corrupt regime

Consider next what happens if there are low-powered incentives, such that officers and miscreants can agree on a bribe \( b(i, f) \). The potential miscreant faces an expected sanction of \( \hat{x}_{c} = pb(i, f) \), which determines a crime level \( 1 - G(pb(i, f)) \). Since enforcement functions through bribe exchanges, the government neither expects to pay out incentives nor receive fine income. Welfare is thus determined by the crime rate and the payment of basic salaries only,

\[
W_c(p, w, i, f) = \int_{pb(i, f)}^{\infty} (x - h)g(x)dx - pw. \tag{6}
\]

The government maximises this welfare function subject to \( b(i, f) < i \), the limited liability constraint (LL) and the following participation constraint:

\[
w + b(i, f)[1 - G(pb(i, f))] \geq \bar{v}, \tag{PC_c}
\]
with either (LL) or (PCc) or both binding at the optimum (as proven later). Incentives are never paid out, but from (6) we can immediately show that they play two roles. First, they raise the equilibrium bribe and, therefore, increase the expected sanction and consequently the compliance threshold. Second, incentives here have the same enforcement function as fines $f$ in a standard beckerian enforcement setting. The following result should, therefore, be quite unsurprising:

**Remark 3** The optimal enforcement policy in the corrupt regime includes a policy with low powered incentives $i < i^{nc}$ such that $b = \sigma f$ and $f = \bar{f}$ (fines and bribes are maximal). Thus, $\hat{x}_c = \sigma p \bar{f}$.

The full enforcement policy will be characterized and proved later. Suffice here to offer a few observations. Note that deterrence from corrupt officers is lower than deterrence from non-corruptible ones: for a given number of police officers $p$, $\hat{x}_c = \sigma p \bar{f} < \hat{x}_{nc} = p \bar{f}$. This is a clearly negative aspect of corruption-based enforcement. However, there are two countervailing factors that make corruption more desirable. First, bribes can reduce the size of the budget by reducing wages that are paid by the government. Importantly, the higher the $\sigma$, the lower the base wage $w$ that needs to be offered to officers. Secondly, an enforcement system based on bribes avoids deadweight losses associated with the collection of fines. The lower $\alpha$ is, the larger these deadweight losses are and, as a consequence, the larger is the advantage of corruption.

To see how these different elements play against one another, define the cost of corruption $C_c(\hat{x})$ to be the difference in the maximum welfare between the corrupt and the benchmark case, for enforcement level $\sigma p \bar{f} = \hat{x}$, subject to the constraints (LL) and (PCc):

$$C_c(\hat{x}) \equiv W_{honest}(\hat{x}) - W_c(\hat{x}) = \begin{cases} (1 - \sigma)pv - (1 - \alpha)\sigma p \bar{f}[1 - G(\hat{x})] & \text{for } v \geq \tilde{v}_\sigma' \\ \sigma p \{\alpha \bar{f}[1 - G(\hat{x})] - v\} & \text{for } v < \tilde{v}_\sigma' \end{cases}$$
where \( \tilde{v}'_\sigma \equiv \sigma \bar{f}[1 - G(\sigma p \bar{f})] \) is increasing in \( \sigma \), as for the case of the clean regime.

We plot the \( C_c(\hat{x}) \) function in the bottom panel of figure 1 for two values of \( \sigma \). \( C_c(\hat{x}) \) can be either positive or negative, depending on the value of \( \sigma \) and \( v \). A corrupt system has lower deterrence and this hurts welfare when a country is rich \((v \geq \tilde{v}'_\sigma)\): owing to the reduction in deterrence, the government needs to hire (and pay) many more officers to achieve the desired level of enforcement. Clearly, this increases the cost of enforcement relative to the benchmark, especially when tolerance for corruption \( \sigma \) is low, and bribes are not effective deterring tools. As market wages fall, the cost of corruption falls, and we obtain the familiar observation that, in many instances, an enforcement system based on bribes is efficient and superior to whatever is achievable by an honest regime. A corrupt regime simply pays its own officers less, in exchange for allowing them to keep their bribes.

Once market wages fall even further below \( \tilde{v}'_\sigma \), additional reductions in \( v \) increase the cost of corruption. This is because at low market wages enforcement is no longer affected by reduction in \( v \), whereas honest enforcement continues to become cheaper and cheaper. Meanwhile, all the potential revenues accrued in the honest regime are essentially passed to enforcement agents in the corrupt regime. Thus, corruption as an institution is most advantageous when the country has middle income, but is less advantageous or even disadvantageous for the same country at a higher and at a lower poverty level.

**Optimal “corrupt” enforcement** Define two threshold wage levels \( \tilde{v}^c_\sigma \equiv \sigma \bar{f}[1 - G(h)] \) and \( \tilde{v}'_\sigma \equiv \sigma \bar{f}[1 - G(\frac{h}{2})] \). As before, Assumptions 1-3 prevent corner solutions, while relaxing these assumptions would not change our results. The following proposition characterizes the optimal policy under corruption.

**Proposition 4** The optimal interior enforcement policy under low-powered incentives is as follows:

a. When \( v > \tilde{v}'_\sigma \), [LL] is slack and [PCc] binds, and the interior optimal enforcement policy \( \hat{x}_c \) solves \( \hat{x}_c = \frac{1}{2}[h + \frac{1}{\lambda} - \frac{\bar{v}}{\sigma f \bar{g}(\hat{x}_c)}] \).
b. When \( v^{\ast}_{\sigma} \leq v \leq \tilde{v}^{\ast}_{\sigma} \), (LL) and (PCc) both bind with equality, and the enforcement level is
\[
\hat{x}_c = G^{-1}(1 - \frac{v}{\sigma f}).
\]

c. When \( v < v^{\ast}_{\sigma} \), (LL) binds, (PCc) is slack, and the optimal level of enforcement is \( \hat{x}_c = h \).

Proof. See appendix. ■

When \( v > \tilde{v}^{\ast}_{\sigma} \) (case a.), the government is able to reduce wages below the market level, such that gains from bribes are exactly offset by the lower base pay. Officers do not earn rents. As market wages fall further below the threshold \( \tilde{v}^{\ast}_c \) (case b.), the government cannot reduce wages further, but can still limit the expected gains of officers by hiring more officers, which reduces the number of miscreants, and thus reduces the probability of a bribe exchange. Thus, officers continue to earn no surplus from bribe taking. As market wages fall below \( v = v^{\ast}_c \), enforcement reaches a maximum \( \hat{x} = h \). At this level, the government is unwilling to increase enforcement further, so any further reduction in market wages translate into positive rents to officials.

Two additional observations are in order. First, it is tempting to think that the maximum amount of enforcement under the corrupt regime, \( \hat{x} = h \), is the “optimal” amount of enforcement. But this observation would only be correct in a frictionless world.\(^{16}\) Second, it is not possible to derive any consistent result about the amount of over-or under-enforcement in the corrupt regime, relative to the benchmark case; the comparison is very sensitive to differences in \( \sigma \) and \( \alpha \).

As in the clean regime, it is possible to have multiple optimal policies, some of which do not require maximal fines or maximal bribes. However, all these policies are qualitatively equivalent to one presented in our proposition. It is without loss of generality that we specialize to the case where fines and bribes are maximal, but the proof includes all others

\(^{16}\)In a frictionless world where there are no inefficiencies from transferring resources though a government budget \( B \), the only concern for the government is the reduction of the social harm. An enforcement of \( \hat{x} = h \) guarantees that only those whose private benefit \( x \) exceed the social harm \( h \) decide to engage in the harmful activity. However, the world we model is not frictionless. Our government has a revenue concern, where revenues are useful to reduce deadweight losses originating from other government expenditures in the larger government budget.
for completeness.

### 3.3 Optimal regime choice in the absence of legalization

The government can move from a corrupt to a clean regime by modifying the incentive pay $i$ so as to meet or violate the no-corruption compatibility constraint $[IC]$. The final outcome is determined by maximizing both $W_{nc}$ and $W_c$, and then choosing whichever is larger. When choosing which regime to adopt, the government essentially trades off lower wages in a corrupt regime, with the revenue that is collected when the regime is clean and the higher per-officer deterrence. Ceteribus paribus, a high-$\sigma$ country will be more likely to choose corruption while a low-$\sigma$ country will be more likely to prevent corruption. Armed with this intuition, we first define the level $\sigma$ such that the government is indifferent between the two regimes, and then prove that, for any parameter value, low $\sigma$ (i.e., below the threshold) involve no corruption, and high $\sigma$ (above the threshold) involve corruption. We then describe how this threshold changes across parameter values.

**Definition 5** Let $\sigma^*$ be the level of $\sigma$ such that, for given parameter values $h, v,  \tilde{f}, \alpha$, the optimal policies in both regimes are such that $W_{nc}^* = W_c^*$.

**Proposition 6** For all parameter values with interior enforcement level:

i. There exists a unique $\sigma^*$ such that when $\sigma \leq \sigma^*$ the regime is clean (policy follows proposition $[\hat{F}]$), and when $\sigma > \sigma^*$ the regime is corrupt (policy follows proposition $[\hat{A}]$).

ii. $\sigma^* \geq \alpha$, with equality iff $v \leq \alpha \tilde{f} \left[1 - G\left(\frac{h}{1+\alpha}\right)\right]$.

We sketch the intuition here and address the reader to the appendix for the complete proof. In the first part, we establish in a lemma that when $\sigma < \alpha$, the regime is always clean ($W_{nc}^* > W_c^*$), and when $\sigma = 1$ the regime is always corrupt ($W_{nc}^* < W_c^*$). Second, we show that $W_{nc}^*$ is continuous and nonincreasing in $\sigma$, and that $W_c^*$ is continuous and nondecreasing in $\sigma$. Those facts establish the existence of $\sigma^*$: the two welfare functions need to cross at
some intermediate $\sigma^*$. To demonstrate uniqueness, we show that $\frac{\partial W^*_j}{\partial \sigma} \geq 0$ and $\frac{\partial W^*_{nc}}{\partial \sigma} \leq 0$; moreover, whenever the j-regime welfare $W^*_j$ does not vary with $\sigma$ (i.e., $\frac{\partial W^*_j}{\partial \sigma} = 0$), the other is strictly increasing or decreasing with $\sigma$. For part (ii.), we look specifically at an economy where $\sigma = \alpha$, and show that when $v \leq \bar{v}_{nc}^\sigma$, $W^*_c = W^*_{nc}$, and when $v > \bar{v}_{nc}^\sigma$, $W^*_c < W^*_{nc}$.

The proposition is best understood by looking at figure 2, which plots the optimized welfare functions in both regimes on the $(\sigma, v)$ planes (details on the simulation in the appendix B). Welfare is negatively related to $v$ in both regimes, a consequence of the fact that increases in $v$ increase the budget devoted to enforcement. As expected, welfare in the clean regime is negatively associated with $\sigma$, but only at low levels of income $v$, whereas welfare in the corrupt regime is positively associated with $\sigma$ at high levels of $v$. It is especially clear that welfare is very sensitive to changes in $\sigma$ for the corrupt regime, because in this regime $\sigma$ directly affects deterrence. $\sigma^*$ is defined by the intersection of the two planes.

Figure 3 more clearly traces $\sigma^*$ for two different levels of $h$, i.e., for activities of different severity. For low levels of $v$, we have that $\sigma^* = \alpha$. Note that this condition implies that corruption is preferred for all cases where $i^{nc} = \sigma f > \alpha f$, where enforcement incentive costs are larger than revenues. The only cases where officers will remain clean are in cases where $\sigma < \alpha$, that is, where revenues from enforcement outweigh incentive costs. Thus, revenue-rich enforcement activities like tax audits should always remain clean, whereas arguably “low revenue” activities like pollution control would remain corrupt. Note that, as $v$ increases, $\sigma^*$ also begins to increase above $\alpha$, and the clean regime becomes preferable for a wider set of parameters. The point where this happens is when $v = \alpha f[1 - G(\frac{h}{1+\alpha})]$, which correspond to the market wage where the limited liability condition (LL) ceases to be binding in the clean regime. Above this market wage, the clean regime becomes generally more preferable because the government does not need to introduce any distortions to manage officers’ corruptibility. In addition, as each officer becomes more expensive, the government values more the higher deterrence level achieved in a clean regime.
Note that the $\sigma^*$ also responds to the severity of the misdeed: increasing $h$ pushes the $\sigma^*$ threshold to the left, such that officers are less likely to be corrupt for more severe crimes. This is because, when the severity of the crime increases, the optimal response in both clean and corrupt regime is to increase the amount of enforcement. With higher enforcement comes a lower level of harmful behavior. This reduces social harm on the one hand, and also reduces the share of total officers’ pay derived from incentives or bribes. In turn, this slackens the $(LL)$ constraints for all regimes, such that $v_j^\sigma$ and $\tilde{v}_j^\sigma$ shift to the left.

Proposition 6 and figure 3 thus suggests what is well known in the literature: the corrupt regime is more likely to be preferable in poor countries, where governments face anti-corruption policies that are too expensive to implement. However, recall that, at low levels of $v$, both corrupt and clean regime perform badly against the benchmark regime. Thus, the fact that at low $v$ enforcement agents are corrupt is less a sign of efficiency, and more of a choice between two bad options.

4 Tax and Legalize

We now introduce the tax-and-legalize mechanism. The government now makes the harmful activity legal, but requires the payment of a reduced fine or tax $r$. It now employs a police force to monitor the payment of the tax and assess fines $f$ as the punishment for tax evasion.

The timing of the model is modified as follows: the government chooses an enforcement policy $f, p, w, r$. Individuals then simultaneously choose whether to commit the harmful act and, if so, whether to pay the tax $r$. Audits follow as usual, except that the officer now must only determine whether an act has been committed without the tax payment. If there was tax avoidance, the officer can assess the fine $f$ or accept a bribe $b$.

It is reasonable to assume that the bureaucratic process for processing taxes would follow a different path than a sanctioned fine. A citizen who is reported to the judiciary by a law enforcer has the right to defend himself in a court of law, and demonstrating his culpability is time consuming and expensive. The same cannot be said for processing a taxpayer. As a
consequence, the bureaucratic procedure needed to process the tax is more efficient and less expensive. We again abstract from this point already highlighted by Kaplow and Shavell (1994) to pinpoint a different mechanism that is not dependent on a different bureaucratic efficiency of self reporting. We thus assume that the tax $r$ is subject to the same efficiency losses as other fines, and the government retains only $\alpha r$ of the paid tax.

In a legalized regime, an individual who commits a harmful act can either pay the tax $r$, or avoid the tax and risk being caught - whichever is more convenient (less expensive) in expectation to him. The punishment $f_j$ is equal to $f$ in the clean regime ($j = nc$) or the bribe $b$ in the corrupt regime ($j = c$). An individual with private gain $x$ may commit the act if his private benefit from the act exceeds the cost:

$$x_j \geq \min[r, pf_j] = \tilde{x}_j^l,$$

where $r$ is now part of the set of policy instruments available to the government. To get the optimal level of $r$, consider first the case $r > pf_j$. Because the tax is more expensive than the expected full sanction, perpetrators avoid the tax. In this case, the tax is ineffective, and no one employs it.

Now suppose $r \leq pf_j$. Clearly, everyone pays the tax. Since $\tilde{x}_j^l = r$, the total number of crimes committed is $1 - G(r)$. Since officers never find a tax evader, they never earn bribes or incentives, and their only source of income (and only source of enforcement cost) is the base wage $w$. Thus, the welfare achieved is

$$W_j^f(r, p, w, i, f) = \int_r^\infty (x - h)g(x)dx - pw + \alpha r[1 - G(r)], \quad (7)$$

subject to the usual constraints [PC] and [LL].

Proposition 7 explains the optimal tax-and-legalize policy.

**Proposition 7** When the tax-and-legalize policy is adopted, all perpetrators pay the tax and:

(i) $i \geq \sigma f$ (officers are always honest);
(ii) \( r = pf \) (the tax is equal to the expected punishment);

(iii) \( w = v \) (officers’ base wage is equal to their reservation wage).

**Proof.** see the Appendix. ■

In equilibrium, incentives are sufficiently high to ensure that officers are honest, but are never paid out because every perpetrator chooses to pay the tax. Even though officers never uncover evaders, the presence of officers and the incentives they could receive do have a role: they act as a credible threat against tax evasion, and guarantee the officers’ honesty when offered bribes. This mechanism resembles the ‘threat’ found in modern income tax systems, or the enforcement of parking regulations: cities allow drivers to park in certain areas only if they pay a tax at the curb by feeding a meter. People feed the meter in the off chance that a parking inspector passes by and fines those who did not.

With this proposition, maximized welfare is a function of fines and enforcement costs,

\[
W_i(pf) = \int_{pf}^{\infty} (x - h)g(x)dx - pv + p\alpha f[1 - G(pf)].
\]  (8)

It is readily observed that this is the same welfare function achieved when officers are honest, \( W_{\text{honest}}(pf) \), equation (3). This implies that fines are maximal at \( \bar{f} \) and the optimal size of the office corp is the same under the tax-and-legalize regime as in the benchmark case, that is, dictated by the first order conditions (4). We highlight this in the following remark:

**Remark 8** The tax-and-legalize regime achieves the benchmark policy and welfare.

### 4.1 Optimal regime choice with legalization

Having determined that the tax-and-legalize regime achieves the same welfare as the benchmark regime, we can use the results from section 3.1 to readily compare a regime cum legalization with a clean regime without it. First, legalization does not improve welfare if a country is sufficiently rich such that (LL) is slack (i.e., for \( v > \tilde{v}_{nc}^{\sigma} \)). At lower levels of wealth
(v ≤ \tilde{v}_{\alpha}^{\text{nc}}), welfare in the clean regime is limited by the additional (LL) constraint, which is absent in the tax-and-legalize regime. Legalization thus offers strictly higher welfare.

Having established that the tax-and-legalize regime replaces the clean regime at low levels of wealth, the following proposition shows the condition under which the tax-and-legalize regime also replaces the corrupt regime:

**Proposition 9** Legalization reduces the parameter region where corruption is optimal iff the country is sufficiently poor (v < \tilde{v}_{\alpha}^{\text{nc}}). In addition, it always replaces corruption when the country is very poor (v ≤ \alpha \bar{f}[1 − G(h)]), regardless of how corruption tolerant the country is.

**Proof.** See appendix. ■

Figures 4, 5 and 6 illustrate the welfare and the various regime choices with and without legalization. Figure 4 plots the welfare levels for each of the three regimes on the (v, \sigma) axis. The blue lines in figures 5 and 6 plot the \sigma^* that divides the clean and the corrupt regime, while the green line identifies the “new” \sigma^* with legalization. It is clear that legalization is implemented both in previously clean and corrupt regimes. However, the parameter space taken by the tax-and-legalize regime is more likely to include previously corrupted regimes when \alpha is relatively small. Thus, this policy is more important for poor, inefficient and corrupt countries.

A final point that is relevant is that the size of the region taken up by the tax-and-legalize regime falls the larger the social cost h: legalization should be more commonly used to regulate activities that are not too damaging.

### 4.2 Social harm under legalization

We have seen that legalization improves social welfare in those parameter regions where corruption is most costly—at low levels of wealth. How does legalization affect enforcement levels and, consequently, the amount of social harm suffered? In the previous sections we established that the benchmark regime (which is the legalized regime) might feature lower
enforcement than both the clean and corrupt regime. This is important because a common concern with legalization is that it may increase the amount of harmful activity. In what follows, we consider in turn the optimal amount of enforcement under legalization relative to $\hat{x}_{nc}$ and $\hat{x}_c$, and will generally find that, in most areas where one expects to find legalization, more harm indeed must be allowed.

**Harm relative to the clean regime:** As discussed in section 3.1, optimal enforcement under legalization has generally higher levels of socially harmful activity when measured against the clean regime. More formally:

**Remark 10** Relative to optimal enforcement in the clean regime, legalization leads to lower enforcement for wealth values $(1 - \lambda h)\gamma^{nc}_\sigma < v < \bar{v}^{nc}_\sigma$, and higher enforcement for wealth values $v < (1 - \lambda h)\gamma^{nc}_\sigma$, where $\gamma^{nc}_\sigma = \bar{w}^{nc}_\sigma / (1 + \alpha - \sigma)$. When the social harm is sufficiently serious ($h > 1/\lambda$), it is always the case that legalization leads to an increase in the amount of the harmful act.

The proof is relegated in Appendix B (not for publication.) This is a striking result: in most cases, whenever it is socially beneficial to switch from a clean regime without legalization to a tax-and-legalize regime, the amount of harm allowed increases. This is driven by the fact that, when $v < \bar{v}^{nc}_\sigma$, the optimal response in the clean regime is to over enforce, and that over-enforcement is corrected by legalization. In particular, as $v$ declines below $\bar{v}^{nc}_\sigma$, over-enforcement increases, reaches its maximum at $v = \bar{x}^{nc}_\sigma$, and then shrinks again because the enforcement level in the clean regime $\hat{x}_{nc}$ remains constant while the enforcement level in the tax-and-legalize regime keeps increasing as $v$ decreases. At the wealth level $v = (1 - \lambda h)\gamma^{nc}_\sigma$, we have that the enforcement level are identical. While legalization leads to enhanced enforcement when market wages are below this threshold, it should be noted that the threshold is negative (and therefore never reached) when $\lambda h > 1$, that is, when the harm from the sanctioned activity is sufficiently serious. Thus, the parameter space where under-enforcement happens can be limited.
**Harm relative to the corrupt regime:** The discussion of enforcement under legalization relative to the corrupt regime is not as clear cut: depending on parameter values, self reporting could have higher or lower levels of enforcement, so the amount of the sanctioned activity might be higher or lower. We thus focus our attention to the parameter space where legalization most clearly allows a switch of regime from corrupt to clean, i.e., when $v < \alpha f[1 - G(h)]$ as discussed in Proposition 9.

**Remark 11** When $v < \alpha f[1 - G(h)]$, self reporting has lower enforcement levels than the corrupt regimes for wealth values $v > (1 - \lambda h)\gamma_c^c$ and higher enforcement for wealth values $v < (1 - \lambda h)\gamma_c^c$, where $\gamma_c^c = \psi^c_\sigma \alpha/\sigma$. When the social harm is sufficiently serious ($h > 1/\lambda$), there is more harm under self reporting.

The proof is relegated in Appendix B (not for publication.) The remark suggests that, in many relevant cases where you expect a switch from corruption to a clean regime with legalization, the shift causes the amount of harm to increase. As in the case for the clean regime, this is due to the fact that there is over-enforcement in the corrupt regime, which is corrected by legalization. The remark also identifies the condition on wealth below which legalization has more enforcement. As for the clean regime case, this condition $v < (1 - \lambda h)\gamma_c^c$ is never met when the harm is sufficiently large ($\lambda h > 1$). If the activity is very harmful $h > 1/\lambda$, legalization always increases the amount of harm.

Unfortunately, we cannot derive any additional conclusion on enforcement levels for values of larger values of wealth (i.e., $v > \psi^c_\sigma$) where there might also be switches from a corrupt regime to the legalized regime. To provide some intuition of what happens there, we simulate the model and draw the the optimal $\hat{x}_j$ on the $(v, \sigma)$ plane for a high-harm activity ($h > 1/\lambda$) in figure 7 and 8. The figures compare enforcement in the clean and corrupt case against the legalized regime. It is quite clear that enforcement is substantially lower under legalization in the parameter regions where legalization is socially optimal. In addition, the area where the corrupt regime has a higher enforcement level than the legalized regime is significantly larger than the area identified in remark 11. On the other hand, in figures 9 and 10, we show
that for a low-harm activity \( h < 1/\lambda \) there is a region where, as expected, self reporting has higher enforcement levels.

In conclusion, our model shows that legalization will generally increase harm, but that this increase is optimal for society. But even in our framework it is not hard to see that this aspect might be problematic. In a world where utility is not perfectly transferable, the harm suffered from certain individuals might not be fully refundable. Legalization could easily spiral into a controversial and politically unpalatable choice. This might explain why legalization is not adopted in the real world as frequently as our model suggest.

It is important to stress that the decrease in enforcement brought by legalization is due to the changes in the incentive costs faced by the government, and it has nothing to do with either changes in consumer taste or to introduction of a revenue concern for the government brought by legalization. This is because in our model legalization does not change the demand for the illegal good, and the government always faces a revenue concern–even in the absence of legalization\(^{17}\).

5 Conclusion

In this paper, we studied the effects of corruption on law enforcement policies and showed that these effects vary with the country’s wealth, tolerance for corruption, and bureaucratic inefficiency. In our model, law enforcers not only punish miscreants who are caught, but also act as a deterrent against committing the harmful act to begin with. The main drawback of corruption is that it dilutes deterrence, by deducting the cost associated with committing a sanctioned act. As in much of the economic literature on corruption, however, we find that the practice is indeed preferable in many cases: cleansing enforcement officials of corruption can be expensive, and the alternative–an honest but inefficient bureaucracy–may not be very desirable. In particular, the poorer the country, the less likely it is going to be that a

\(^{17}\)There is significant empirical evidence that legalization (or, conversely, prohibition) of an illegal activity changes the demand for a good. See footnote \(^{1}\). In our model, we could easily introduce this feature by having a measure of infinitely risk averse citizens. Self reporting would immediately result in a demand-side response, with higher demand for the harmful good at all levels of enforcement. Notes available upon request.
government will adopt sufficiently stringent anti-corruption policies needed to keep officers honest. While corruption is indeed socially preferable at low income levels, it is also at those levels that the practice is at its most costly. We highlight this by showing high welfare differentials against equivalent countries with same level of structural parameters but an inability for officers to receive bribes. At intermediate income levels, corruption may be efficient: it provides the right combination of deterrence, low enforcement costs, and lack of bureaucracy. At higher income levels, corruption ceases to be either a resource or a problem.

In this rich but simple model of corruption, we find that a policy of legalization changes the incentives faced by the policy makers in such a way that it reduces the situations in which corruption arises and eliminates the costs associated with the threat of corruption. More importantly, legalization is always preferable at the lowest income levels—when corruption is most likely to occur and where corruption incurs the highest costs.

This stark result from the model raises an important question: Why is legalization/self reporting not more widespread in poor countries? Possibly because legalization achieves this higher welfare by increasing the amount of socially harmful activities. In a political economy setting where a policy is chosen not only on its impact on welfare, but also on the political viability of the policy, legalization could very well be controversial. For instance, if citizens are more likely to observe (and vote on) the amount of social harm as opposed to the amount of taxation, legalization would be a politically unpalatable choice. Alternatively, if the impact of the harmful activity is non-uniform, the emergence of interest groups becomes plausible.
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6 Appendix

6.1 Proof of proposition 2

We first prove statements a. and c., and then use the results to prove statement b. The proofs demonstrate that the optimal set includes a maximal $f$ and $i = i^{nc}$. See web Appendix B for a proof with a characterization of all optimal policies (including those where $f < \hat{f}$, and $i > i^{nc}$).

**Part a.** Consider first the case where $\text{[LL]}$ is slack. Then, $\text{[PC}_{nc}]$ binds and, from $\text{[PC}_{nc}]$, we get that the optimal wage is $w_{nc} = v - i[1 - G(pf)]$. Replacing this in the welfare function (5), we get that

$$W_{nc}(p, f) = \int_{pf}^{\infty} (x - h)g(x)dx - pv + pf[1 - G(pf)].$$

(9)

Note first that welfare does not depend on $i$, and any $i \geq \sigma f$ satisfies $\text{[IC]}$. In addition, the welfare is equivalent to (3), the benchmark case without the possibility of corruption. The optimal policy thus follows the benchmark case, which means that $f$ is maximal and the optimal interior solution $p_{nc}$ is determined by the first order condition (4).

We next determine the optimal $\hat{i}^*$. As long as $v > \hat{i}^*[1 - G(p_{nc}\hat{f})]$, $\text{[LL]}$ is slack. Thus, any $i^* \in [\sigma f, \frac{v}{1 - G(p_{nc}\hat{f})}]$ ensures that $\text{[LL]}$ does not bind. Next, we show the parameter space where this solution is feasible. The largest $v$ such that $\text{[LL]}$ binds is $v = i^*[1 - G(p_{nc}\hat{f})]$; substituting this in (4) we get

$$h - (1 + \alpha)p_{nc}\hat{f} = \frac{i^*[1 - G(p_{nc}\hat{f})]}{f g(p_{nc}\hat{f})} - \frac{\alpha}{\lambda}.$$  

(10)

Solving for $p_{nc}\hat{f}$ and considering that the hazard rate $\frac{1 - G(p_{nc}\hat{f})}{g(p_{nc}\hat{f})} = \frac{1}{\lambda}$, we get $p_{nc}\hat{f} = h(1 + \alpha) - i^*\frac{\alpha f}{f\lambda(1 + \alpha)}$. Substituting $p_{nc}\hat{f}$ and $i^* = \sigma\hat{f}$ into the participation constraint, we find that this solution is feasible for $v > \tilde{\sigma}_{\alpha}^nc \equiv \sigma\hat{f} \left[1 - G\left(\frac{h}{1 + \alpha} - \frac{\sigma - \alpha}{\lambda(1 + \alpha)}\right)\right]$. It remains to be demonstrated that, in the parameter space where the solution is feasible, it is also optimal. This follows immediately from the fact that when $v > \tilde{\sigma}_{\alpha}^nc$, the government achieves benchmark welfare, and the benchmark welfare is strictly better than any clean regime welfare where $\text{[LL]}$ binds. Thus, it is never optimal to set $i > \sigma\hat{f}$ in such a way that $\text{[LL]}$ binds. Moreover Assumptions 1-3 ensure no corner solutions.

**Part c.** Next consider the case where $\text{[LL]}$ binds and $\text{[PC}_{nc}]$ does not. Substituting $w_{nc} = 0$ in the welfare function (5), we get an equation that is decreasing in $i$. Thus, $i$ should be reduced until $\text{[IC]}$ binds: $i^{nc} = \sigma\hat{f}$. The welfare function takes the form

$$W_{nc}(p, f) = \int_{pf}^{\infty} (x - h)g(x)dx + pf(\alpha - \sigma)[1 - G(pf)].$$

(11)

This function does not separately identify $p$ from $f$: rather, it identifies $\hat{x}_{nc}$, and then any combination $p$ and $f$ such that $pf = \hat{x}_{nc}$ is equally acceptable. Here, we focus on $f = \hat{f}$. See web appendix for a proof with a characterization of all optimal policies. Setting the derivative of $W_{nc}$ with respect to $p$ equal to zero we get

\[18\] An interior solution is guaranteed if $v < \hat{f}(h\lambda + \alpha)$ (always satisfied by assumption A2 and by assumption A1, which implies that $h > (\sigma - \alpha)/\lambda$ and if $v > f g(f) \left(h - \hat{f}[1 + \alpha] + \frac{\sigma}{\lambda} \right) = \lambda h - (1 + \alpha)f + \alpha/\lambda] f[1 - G(f)]$, which is always satisfied because the threshold is smaller than $\tilde{\sigma}_{\alpha}^nc$ and by assumption A1.
\[ h - (1 + \alpha - \sigma)p_{nc}\tilde{f}g(p_{nc}\tilde{f}) = (\alpha - \sigma)[1 - G(p_{nc}\tilde{f})]; \tag{12} \]

Dividing both sides by \( g(p_{nc}\tilde{f}) \) and substituting \( 1 - G(p_{nc}\tilde{f}) \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \). An interior solution for \( p \) is guaranteed by assumption A1 and by \( h > \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \). There is an interior solution.

Finally, this solution applies whenever \( \{PC_{nc}\} \) is slack; that means that it is feasible in the space \( v < \sigma\tilde{f}\left[1 - G\left(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}\right)\right] \).

Part b. Next consider the case where \( \{PC_{nc}\} \) and \( \{LL\} \) bind at the same time. Substituting \( w_{nc} = 0 \) and the binding \( \{PC_{nc}\} \) in the welfare function \( (5) \), we get an equation that is decreasing in \( i \). Thus, \( i_{nc} = \sigma f \), the welfare function takes the form \( (11) \), and the optimal enforcement policy \( p_{nc} \) and \( f_{nc} \) solves \( v = \sigma f_{nc}[1 - G(p_{nc}f_{nc})] \). Rearranging, we get the optimal interior enforcement solution \( \hat{x}_{nc} = G^{-1}(1 - \frac{v}{\sigma f_{nc}}) \). Plugging this into \( (11) \), we get that the equilibrium welfare is

\[ W_{nc}(p_{nc}, f_{nc}) = \int_{\hat{x}_{nc}}^{\infty} (x - h)g(x)dx + p_{nc}(\alpha - \sigma)\frac{v}{\sigma}. \tag{13} \]

We next show that, if \( v_{\alpha}^{nc} \leq v \leq \tilde{v}_{\alpha}^{nc} \), \( f_{nc} = \tilde{f} \). Suppose \( f < \tilde{f} \), but \( p \) is chosen such that \( v = \sigma f[1 - G(pf)] \). Then, we could increase \( f \) to \( f' > f \) and reduce \( p \) to \( p < p' \) such that \( f'p' = pf \). Note that \( v < \sigma f'[1 - G(p'f')] \), \( \{PC_{nc}\} \) is now slack, and the welfare achieved is determined by equation \( (11) \). Comparing this welfare function with equation \( (13) \), the two are identical:

\[ W_{nc}(p, f) - W_{nc}(p', f') = (\alpha - \sigma)\frac{v}{\sigma} - p'f'(\alpha - \sigma)[1 - G(p'f')] = (\alpha - \sigma)[pf[1 - G(pf)] - p'f'[1 - G(p'f')]] = 0. \]

However, unlike \( W_{nc}(p, f) \), \( W_{nc}(p', f') \) is generally not evaluated at its optimal expected sanction. The optimal expected sanction \( \hat{x}_{nc} = p'f' \) is \( p'f' = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \) (see Part c. of this proof). The only time when \( pf = p'f' \) and the two welfare functions are equal is when \( v = \sigma f\left[1 - G\left(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}\right)\right] < v_{\alpha}^{nc} \). But here we are considering the case where \( v \geq v_{\alpha}^{nc} \). So it must be that the welfare is higher with \( f' \). This contradicts the assertion that \( f \) was optimal\(^{19} \).

We conclude by noting that this solution is feasible for the space \( v_{\alpha}^{nc} \leq v \leq \tilde{v}_{\alpha}^{nc} \). It can be verified that at the boundary \( v = v_{\alpha}^{nc} \), the solution is \( p_{nc}\tilde{f} = \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \), which is the same solution found in Part c. Also, at the boundary \( v = \tilde{v}_{\alpha}^{nc} \), the solution is \( p_{nc}\tilde{f} = \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha)} \), which is the solution for Part a. Outside of this interval, the solution in Part b. is dominated by the solutions in Part c. and Part a. Thus, the solution in Part b. applies to the space \( v_{\alpha}^{nc} \leq v \leq \tilde{v}_{\alpha}^{nc} \), and \( W_{nc}^{*} \) is continuous in \( v \).

### 6.2 Proof of proposition 4

Here we prove that in the corrupt regime maximal \( b \) and \( f \) (i.e., \( b = \sigma \tilde{f} \) and \( f = \tilde{f} \)) are always in the optimal policy set, and analyze this solution only. The complete proof that solves for all possible policies can be found in the web Appendix B.

\(^{19}\) An interior solution requires \( v > \sigma f[1 - G(\tilde{f})] \), which is satisfied by \( v > v_{\alpha}^{nc} \) and by assumption A1, and \( v < \sigma \tilde{f} \) (assumption A2).
Part a. Consider first the case where, for a given value of \( p, b \) and \( w \), a corrupt regime has \( b < \sigma f \), and (LL) does not bind. First, note that the participation constraint must be binding with equality. Otherwise, the government can reduce \( w \) without changing either \( b \) or \( p \), and the wage bill is reduced in a way that does not impact the amount of harm. Thus, \( w_c = v - b(i, f)[1 - G(pb(i, f))] \).

Next, a small increase in \( i \) induces the equilibrium bribe \( b \) to increase to \( \bar{b} \) according to (1), and the participation constraint \( (PC_c) \) slackens. The government can then choose to hire fewer officers \( \bar{p} \) such that \( \bar{p}b = pb \), and keep the pay \( w \). With fewer officers, the wage bill is reduced without affecting the level of harm. Thus, for any wage \( w \) that satisfies \( (PC_c) \), \( i \) is set to a value such that bribes are maximal: \( b = \sigma f \). Since fines also increase the size of the bribe, by the same argument the fines are also maximal: \( f = \bar{f} \). Substituting this into the welfare function (6), we get

\[
W_c(p) = \int_{\sigma pf}^{\infty} (x - h)g(x)dx - pv + \sigma pf[1 - G(\sigma pf)].
\]  

To find the optimal force size \( p_c \), take the derivative of the welfare function with respect to \( p \):

\[
\frac{\partial W_c}{\partial p} = (h - \sigma pf)g(\sigma pf)\sigma f - \tilde{v} - \sigma pfg(\sigma pf)\sigma f + \sigma f[1 - G(\sigma pf)]
\]  

(15)

An interior solution must satisfy the first-order condition that \( \frac{\partial W_c}{\partial p} = 0 \). After rearranging and using the fact that \( \frac{1 - G(\sigma pf)}{g(\sigma pf)} = \frac{1}{\lambda} \), the optimal interior solution \( p_c \) solves \( p_c\bar{f} = \frac{1}{\lambda} \left[ h + \frac{1}{\lambda} - \frac{v}{\sigma f g(\sigma pf)} \right] \).

This is an implicit function; since the left hand side is increasing in \( p \), the solution is unique and it exists. Finally, we determine the parameter space such that the limited liability does not bind. (LL) is slack if \( v > \sigma f[1 - G(\sigma pf)] \). Substituting this in (15) and rearranging, we get the threshold level of enforcement, \( \sigma pf = \frac{b}{2} \). Hence, for any \( v > \tilde{v}_c \equiv \sigma f[1 - G(\frac{b}{2})] \), (LL) is slack, (PC_c) binds and the solution discussed in this part of the proof applies.

Part c. Next consider the case where (LL) binds and (PC_c) does not. Substituting \( w_c = 0 \) in the welfare function (6), we get that the welfare function is

\[
W_c(p) = \int_{\sigma pb(i, f)}^{\infty} (x - h)g(x)dx,
\]  

(16)

which is maximized when \( \hat{x}_c = p_c b_c(i, f) = h \).\(^{22}\) Note that maximal \( b_c = \sigma f_c \) and \( f_c = \tilde{f} \) are part of the solution set. Since (PC_c) does not bind provided that \( v < \sigma f[1 - G(\sigma pf)] \), this solution is feasible for values of \( v < \tilde{v}_c = \sigma f[1 - G(h)] \).

Part b. Consider the case where \( \sigma f[1 - G(h)] \leq v \leq \sigma f[1 - G(\frac{b}{2})] \), such that (LL) and (PC_c) bind at the same time. Then, \( w_c = 0 \) and the welfare function is the same as (16). Enforcement is pinned down by (PC_c): \( p_c b_c(i, f) = \hat{x}_c = G^{-1}(1 - \frac{v}{b_c(i, f)}) \).

---

\(^{20}\) The set of values \( i^* \) such that (i) \( b(i^*) = \sigma f \) and (ii) the regime is corrupt is the interval \( \left[ \max \left(0, \frac{v - \sigma f}{1 - \sigma f} \right), \sigma f \right] \).

\(^{21}\) In the parameter space \( v > \tilde{v}_c \), the solution is always interior because the interior solution requires \( v > \sigma f g(\sigma f) \left[ h - 2\sigma f + \frac{1}{\lambda} \right] = \sigma f \left[ h - 2\sigma f + 1/\lambda \right][1 - G(\sigma f)] \) (which is always smaller than \( \tilde{v}_c \) under assumption A3) and \( v < \sigma f (h\lambda + 1) \), again always guaranteed by assumption A2.

\(^{22}\) By assumption A3, \( \sigma f > h \), the solution \( p_c \) is always interior.
What is left is to determine is the equilibrium bribe and optimal fine. The equilibrium bribe is going to be maximal provided that \( pb(i, f) = \sigma pf < h \) for any value of the parameters in the relevant space \( \sigma f[1 - G(h)] \leq v \leq \sigma f[1 - G(h)] \). That is because welfare (16) is increasing in \( pb(i, f) \) if and only if \( pb(i, f) < h \), and it reaches the maximum when \( pb(i, f) = h \). We first show that, in the parameter space \( \sigma f[1 - G(h)] \leq v \leq \sigma f[1 - G(h)] \) we have \( pb(i, f) < h \), except at \( v = \sigma f[1 - G(h)] \) where \( pb(i, f) = h \) if \( b(i, f) = \sigma f \); hence, for that particular value of \( v \) the government trivially selects the maximal fine and bribe \( b_c(i, f) = \sigma f \). From the participation constraint, we have that

\[
\begin{align*}
p_c b_c(i, f) &= G^{-1} \left( 1 - \frac{v}{b_c(i, f)} \right) \\
&\leq G^{-1} \left( 1 - \frac{\sigma f[1 - G(h)]}{b_c(i, f)} \right) \\
&\leq G^{-1} \left( 1 - \frac{\sigma f[1 - G(h)]}{\sigma f} \right) = h
\end{align*}
\]

where the first inequality is strict iff \( v \neq \sigma f[1 - G(h)] \) and it comes from the fact that the parameter space under consideration has \( v \geq \sigma f[1 - G(h)] \), and the second comes from \( b(i, f) \leq \sigma f \leq \sigma f \). Hence, both the optimal fine and the optimal bribe are maximal, i.e., \( b_c(i, f) = \sigma f \).

We conclude by noting that this solution is feasible for the space \( v_c^* \leq v \leq \tilde{v}_c^* \). In that parameter space, the solution is always interior: the condition for an interior \( p_c \) is \( v > \sigma f[1 - G(\sigma f)] \) and \( v < \sigma f \). The first is always true by assumption A3 and the fact that \( v > v_c^* \), and the second is due to assumption A2. It can also be verified that at the boundary \( v = 2 \tilde{v}_c^* \), the solution is \( \sigma p_c \tilde{f} = \tilde{x}_c = h \), which is the same solution found in Part c. Also, at the boundary \( v = \tilde{v}_c^* \), the solution is \( \sigma p_c \tilde{f} = \tilde{x}_c = \frac{h}{2} \), which is the solution at the boundary for Part a. Outside of this interval, the solution in Part b. is dominated by the solutions in Part c. and Part a. Thus, the solution in Part b. applies to the space \( v_c^* \leq v \leq \tilde{v}_c^* \), and \( W_c^* \) is continuous in \( v \).

### 6.3 Proof of proposition 6

In the following proofs we use the terms benchmark regime and legalized regime as synonyms, because both regimes implement the same optimal policy and welfare. We first introduce a lemma and then use the lemma for the main proof.

**Lemma 12** For interior values of \( p \), the regime is always clean when \( \alpha > \sigma \). When \( \alpha < \sigma = 1 \), the regime is always corrupt.

**Proof.** Intuitively, in a clean regime officers who write a fine receive an additional pay of \( \sigma f \) and they earn \( \alpha f \) to the government coffers. When \( \alpha > \sigma \), the received fine is always larger than the pay, so for any enforcement level the government will prefer to collect the fine by providing high incentives. More formally, consider an optimal corrupt regime \( W_c^* \) with optimal policies \( \tilde{p}, \tilde{f}, \tilde{i}, \tilde{w} \). Set the clean regime to have the following policies: \( f = \tilde{f}, i = \sigma \tilde{f}, p = \sigma \tilde{p}, \) and \( w = \tilde{w} \). The gain in welfare from choosing this alternative policy is given by the difference between the two welfare functions (5) and (6):

\[
W_{nc}(p, \tilde{w}, i, \tilde{f}) - W_c^*(\tilde{p}, \tilde{w}, \tilde{i}, \tilde{f}) = \sigma \tilde{p} \tilde{f}(\alpha - \sigma)[1 - G(\sigma \tilde{p} \tilde{f})] + (1 - \sigma)\tilde{p} \tilde{w} > 0.
\]

We now prove that when \( \alpha < \sigma = 1 \), the regime is always corrupt. When \( \sigma = 1 \), the bribe size is the same as the fine, which means that the corrupt regime offers as good of a deterrent as the
case 2

V > α

Then, prove that

\[ W_c(\tilde{p}, \tilde{w}, i, \tilde{f}) - W_{nc}^*(\tilde{p}, \tilde{w}, i, \tilde{f}) = \tilde{p}\tilde{f}(1 - \alpha)[1 - G(\tilde{p}\tilde{f})] > 0. \]

6.3.1 Part i.

Existence: Inspecting the optimal \( W_{nc}^* \) in (9), (11) and (13), it is clear that \( W_{nc}^* \) is strictly decreasing in \( \sigma \) only when (LL) is binding, while \( W_c^* \) is strictly increasing in \( \sigma \) only when (PC\(_c\)) is binding (see (14) and (16)). Together with lemma 12, these two facts prove existence: at low values of \( \sigma \), the clean regime is preferred and at high values of \( \sigma \) the corrupt regime is chosen, so the two welfare functions need to cross at some intermediate \( \sigma^* \), provided that \( W_{nc}^* \) and \( W_c^* \) are continuous. We thus are left to prove that \( W_{nc}^* \) and \( W_c^* \) are continuous in \( \sigma \). Consider \( W_{nc}^* \). It is clear from inspection of the welfare forms (9), (11) and (13) that they are all continuous in \( \sigma \). In addition, \( W_{nc}^* \) is continuous in \( \sigma \) at the boundaries \( \psi_{nc} \) and \( \tilde{\psi}_{nc} \). The same applies to \( W_c^* \) as defined by (14) and (16), which is also continuous in \( \sigma \) at the boundary thresholds \( \psi_c \) and \( \tilde{\psi}_c \).

Uniqueness: Uniqueness requires that for any parameter value, whenever one of the two welfare functions is constant in \( \sigma \) the other is strictly increasing (or decreasing). The proposition thus immediately follows whenever (LL) binds in the clean regime (\( W_{nc}^* \) strictly decreasing in \( \sigma \)) or (PC\(_c\)) binds in the corrupt regime (\( W_c^* \) strictly decreasing in \( \sigma \)). The only special case arises when (PC\(_c\)) is slack in the corrupt regime and (LL) is slack in the clean regime. The former can happen only if \( v < \psi_c \), while the latter arises only if \( v > \tilde{\psi}_{nc} \). The space where these conditions hold exists iff \( \tilde{h} = \frac{h}{1 + \alpha} - \frac{\sigma - \alpha}{\sigma(1 + \alpha)} > h \), which implies that \( h < -\frac{\sigma - \alpha}{\alpha} \). But \( h > \frac{\sigma - \alpha}{\alpha} \) (because we are excluding \( p_{nc} = 0 \) when \( v < \psi_{nc}^* \)) and lemma 12 implies that we can restrict attention to \( \sigma \geq \alpha \). Thus, there is no parameter space where (PC\(_c\)) is slack in the corrupt regime and (LL) is slack in the clean regime.

Having demonstrated that \( \sigma^* \) exists and is unique, it must be that for values of \( \sigma \geq \sigma^* \) the regime is clean and for values of \( \sigma < \sigma^* \) the regime is corrupt. This follows from the fact that the welfare in the clean regime is decreasing in \( \sigma \) and in the corrupt regime is increasing in \( \sigma \).

6.3.2 Part ii.

We are left to demonstrate that \( \sigma^* = \alpha \) iff \( v \leq \alpha\tilde{f}[1 - G(\frac{h}{1 + \alpha})] \). For this proof, let \( \sigma = \alpha \). First we prove that \( W_{nc}^* = W_c^* \) if \( v \leq \alpha\tilde{f}[1 - G(\frac{h}{1 + \alpha})] \). Then, we show that \( W_{nc}^* > W_c^* \) if \( v > \alpha\tilde{f}[1 - G(\frac{h}{1 + \alpha})] \).

**case 1** \( v \leq \alpha\tilde{f}[1 - G(\frac{h}{1 + \alpha})] \).

In this region, we have that both clean and corrupt regimes have (LL) binding. This is because, at \( \sigma = \alpha \), \( v < \alpha\tilde{f}[1 - G(\frac{h}{1 + \alpha})] = \tilde{\psi}_{nc} < \alpha[1 - G(\frac{h}{2})] = \tilde{\psi}_c \).

Evaluating clean welfare equations (11) and (13) at \( \sigma = \alpha \) we have that \( W_{nc}^* = \int_{x_{nc}}^\infty (x - h)g(x)dx \), which is the same as \( W_c^* \) when \( v < \tilde{\psi}_c \). Thus, \( \sigma^* = \alpha \). By part (i) of the proposition, it must be that this \( \sigma^* \) is unique, and any regime with \( \sigma > \sigma^* \) is corrupt.

**case 2** \( v > \alpha\tilde{f}[1 - G(\frac{h}{2})] \)
Next, consider the case where \( v > \alpha \tilde{f}[1 - G(\frac{h}{1+\alpha})] \), where the right-hand-side of this expression is equal to \( \tilde{v}_c \) when \( \sigma = \alpha \). Both clean and corrupt regimes have \( (L) \) slack. We’ll prove that \( W_{nc}^* > W_c^* \), and this will imply that \( \sigma^* > \alpha \) by part (i) of the proposition. Let \( \hat{x}_c \) be the optimal enforcement under the corrupt regime. Consider the clean policy with (possibly sub-optimal) enforcement policy \( \hat{x}_c \). It must be that \( p_{nc} = \sigma p_c = \alpha p_c \). Then, \( W_c^*(\hat{x}_c) - W_{nc}(\hat{x}_c) = \{\alpha \tilde{f}[1 - G(\hat{x}_c)] - \sigma f[1 - G(\hat{x}_c)]\} p_c(1 - \alpha) < 0 \). The inequality holds because we proved that \( (L) \) is slack in the corrupt regime when \( v > \alpha \tilde{f}[1 - G(\frac{h}{2})] \) and so \( v > \alpha \tilde{f}[1 - G(\hat{x}_c)] \).

**case 3** \( \alpha \tilde{f}[1 - G(\frac{h}{1+\alpha})] < v \leq \alpha \tilde{f}[1 - G(\frac{h}{2})] \)

In this case the \( (L) \) is slack in the clean regime and both \( (L) \) and \( (PC) \) bind in the corrupt regime. (Note that it cannot be that \( (PC) \) is slack, because \( v_c = \alpha \tilde{f}[1 - G(h)] < \alpha \tilde{f}[1 - G(\frac{h}{1+\alpha})] < v \).) We prove that \( W_c^* < W_{nc}^* \) in two steps: In step 1, we show that a clean regime can replicate the (optimal) corrupt regime by choosing the corrupt enforcement level \( \hat{x}_c; \) this establishes \( W_c^* \leq W_{nc}^* \). In step 2, we show that the optimal clean regime will always favor a different enforcement level. This establishes \( W_c^* \neq W_{nc}^* \). **Step 1:** Corrupt welfare is given by equation \( (6) \), \( W_c^*(\hat{x}_c) = \int_{x_c}^{\infty} (x - h)g(x)dx \), where \( \hat{x}_c \) is given by \( v = \sigma f[1 - G(\hat{x}_c)] \). Evaluating the clean regime welfare \( (9) \) at the same enforcement level, we have that \( W_{nc}(\hat{x}_c) = \int_{\hat{x}_c}^{\infty} (x - h)g(x)dx \). Since the optimal enforcement in the clean regime need not be \( \hat{x}_c \), \( W_{nc}^* \geq W_c^* \). **Step 2:** When \( v = \alpha \tilde{f}[1 - G(\frac{h}{1+\alpha})] \), \( W_c^* = W_{nc}^* \). We show that, when we increase \( v \) above this level, enforcement decreases faster under the corrupt regime than under the clean regime, so that \( \hat{x}_c \neq \hat{x}_c \). This implies \( W_c^* \neq W_{nc}^* \). We use total differentiation of \( \hat{x} \) under the two regimes to compute \( \frac{\partial p}{\partial v} \): 

\[
\frac{\partial p_c}{\partial v} = -\frac{1}{fg(pf)} \frac{1}{\sigma f} \]

and 

\[
\frac{\partial pc}{\partial v} = -\frac{1}{fg(pf)} \frac{v}{\sigma f[1 + \alpha]} \]

where, for the latter derivative, we made use of the fact that \( \frac{v}{fg(pf)} = \frac{\sigma f}{\lambda [1 - G(pf)]} \), whose derivative with respect to \( p \) is \( \frac{\sigma f}{1 - G(pf)} \). We thus need to prove that the former derivative is smaller than the latter in the region \( \alpha \tilde{f}[1 - G(\frac{h}{1+\alpha})] < v \leq \alpha \tilde{f}[1 - G(\frac{h}{2})] \). This is so provided that \( \sigma f < f(1 + \alpha) + \frac{v}{1 - G(pf)} \); rearranging, we have the condition \( v > -f(1 + \alpha - \sigma)[1 - G(pf)] \), which is always the case because the right hand side is a negative number.

### 6.4 Proof of Proposition 7

In what follows, we prove the three statements in Proposition 7, i.e., when legalization is adopted, all miscreants self report and the optimal policy entails: (i) \( i \geq \sigma f \); (ii) \( r = pf \), and (iii) \( w = v \).

(i) Suppose that incentives are low, such that the regime is corrupt. We have shown in the main text that the optimal a tax-and-legalize scheme must satisfy \( r_c \leq pb \) in a corrupt regime. The government could provide high-powered incentives \( i \geq \nu \); keep the wage at \( w \) and the tax at \( r_c \), and reduce the workforce from \( p \) to \( p' = r_c/f \). Revenues and enforcement levels would not change; however, the wage bill would be smaller so the self reporting welfare function \( \tilde{W}_c \) would increase by \( W_{nc}(r_c) - W_c^*(r_c) = (p - p')w = (\frac{r_c}{f} - \frac{r_c}{f'})w > 0 \). Thus, legalization is always implemented in a clean regime (i.e., \( i \geq \sigma f \)).
(ii) Suppose \( r < pf \). Then, all criminals self report, and the government could slightly decrease \( p \) without changing the number of crimes. Neither the integral nor the last term in the self reporting welfare function \( (7) \) would change. The second term in \( (7) \) would decrease, because there are fewer wages to pay. Thus, welfare would increase. Since \( r < pf \) is not optimal, it must be that \( r \geq pf \).

If \( r > pf \), no one pays the tax. Thus, it must be that the optimal \( r \) is \( pf \).

(iii) When offenders self report by paying the tax, officers never audit anyone who has committed the sanctioned act and failed to self report. They never earn any incentive pay, and the only salary paid is the base wage. Constraint \( PC \) then reduces to \( w \geq v \). To maximize equation \( (7) \), the government reduces the base wage to the point where the constraint binds, to \( v \).

6.5 Proof of proposition 9

The first part of the proof is straightforward. When \( v > \tilde{v}_{\sigma}^{nc} \), the regime is either clean or corrupt (i.e., legalization implements the same optimal policy and welfare as the clean regime.) Thus, the comparison is driven by proposition 6; thus, \( \sigma^* \) remains the level of \( \sigma \) such that the government is indifferent between the two. When \( v < \tilde{v}_{\sigma}^{nc} \), we have already showed that, compared to a clean regime, legalization implements strictly higher welfare. Hence, the \( \sigma^* \) that separates the self reporting regime from the corrupt regime is going to be a higher value than the one that separates the clean from the corrupt.

For the second part of the proposition, consider \( v < \alpha f[1 - G(h)] < \tilde{v}_{\sigma}^{c} \) and w.l.o.g. set \( \sigma > \alpha \) (we need not prove the case \( \sigma < \alpha \) since by proposition 6, a clean regime trivially dominates the corrupt regime.) The optimal corruption policy is to set \( \tilde{x}_c = \sigma p \hat{f} = h \), which yields a welfare of \( W_c(h) = \int_{h}^{\infty} (x - h) g(x) dx \). Compare that welfare with the welfare derived from the legalization policy \( r = h \) with associated enforcement \( \tilde{x}_c = p \hat{f} = h \). The increase of welfare under this policy is given by

\[
W_l(\tilde{x}_c) - W_c(\tilde{x}_c) = [\alpha f[1 - G(h)] - v] p > 0
\]

where the inequality is satisfied when \( v < \alpha f[1 - G(h)] \) as anticipated.

Consider finally the case \( v = \alpha f[1 - G(h)] \). Our previous result implies that the welfare in the corrupt and legalized regime is the same when they are both at enforcement \( h \). However, from the first order condition \( (4) \), we have that the optimal enforcement level in the legalized regime is not \( h \). Thus, a legalized regime dominates the corrupt regime irrespective of the strength of the institution \( \sigma \), if \( v \leq \alpha f[1 - G(h)] \).
Appendix B: Proofs not intended for publication

7.0.1 Proof of proposition 2: full policy set

We first prove statements a. and c., and then use the results to prove statement b. The proof shows the full set of optimal policies, which include (but are not limited to) policies where \( f \) is maximal.

**Part a.** This part is unchanged from the proof 6.1.

**Part c.** In the proof 6.1 we obtained the solution \( p_{nc}f_{nc} = \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \). Let us next further characterize the solution just identified. This solution applies whenever \( \text{(PC}_{nc}\text{)} \) is slack; that means that \( v < \sigma f_{nc}[1 - G\left(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}\right)] \). Since this condition is an increasing function of \( f \), the set of \( v \) where the solution is feasible is \( v < \sigma \hat{f}[1 - G\left(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}\right)] \equiv \nu^nc \). Note also that there are many combinations of \( f_{nc} \) and \( p_{nc} \) in the parameter space \( v < \nu^nc \) that yield enforcement \( \hat{x}_{nc} \). Specifically, for any \( f \in (v/\sigma[1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])], \hat{f} \), the optimal number of officers is \( p_{nc} \) such that \( p_{nc}f = \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \); under our assumptions this solution is always interior (i.e., \( 0 < p_{nc} < 1 \)). We have left to prove whether or not it is optimal to select a fine \( f \) such that \( f \leq v/\sigma[1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])], \hat{f} \), and, if so, what is the optimal expected penalty \( \hat{x}_{nc} \). We shall study this in the next section.

**Part b.** This part is unchanged from the proof 6.1. Recall that, in the parameter space \( v < \nu^nc \), we have left to show whether or not it is optimal to select a fine \( f \) such that \( f \leq v/\sigma[1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])] \equiv \hat{f} \), and, if so, what is the optimal expected penalty \( \hat{x}_{nc} \). First, consider the case where \( f = \hat{f} \). In this case both \( \text{(PC}_{nc}\) and \( \text{(LL} \) bind. From Part b. follows that \( \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \), and the welfare is the same as in the case where \( f < \hat{f} \) (see Part c.). Hence, the fine \( f = \hat{f} \) is part of the optimal set. Now, take any \( f < \hat{f} \). Part b. implies that for values of \( v \) such that \( v/[\sigma(1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])] \leq f < \hat{f} \) optimality requires that \( \text{(PC}_{nc}\) and \( \text{(LL} \) both bind. Part b. also shows that for these values of \( v \) increasing \( f \) to the point where \( \text{(PC}_{nc}\) is slack, i.e., \( f > \hat{f} \), unambiguously increases welfare. Hence a fine \( f < \hat{f} \) in the interval considered for \( v \) cannot be optimal. Finally, for values of \( v \) such that \( f < v/[\sigma(1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}))] \), Part a. implies that optimality requires only \( \text{(PC}_{nc}\) to bind. Hence, \( v > \sigma f[1 - G(pf)] \), where \( p' \) is the optimal number of officers for a given fine \( f \) and \( v \) in the interval under consideration. Now find a \( f'' \) larger than \( f \) and such that \( f'' \in (v/[\sigma(1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])], \hat{f} \) and a corresponding \( p'' \) such that \( 0 < p'' < p \) and \( p'' f'' = pf' \). Compare the welfare \( W_{nc}(p', f) \) and \( W_{nc}(p'', f'') \), where in the latter case only \( \text{(LL} \) binds by construction, implying that the optimal solution is as in Part c. Note that,\n
\[
W_{nc}(p', f) - W_{nc}(p'', f'') = -p'v + pf(1 - G(pf)) - p'' f''(\alpha - \sigma)[1 - G(p'' f'')] = -p'v + p'f(1 - G(pf')) < 0
\]

where the last inequality comes from noting that in the space under consideration \( v > \sigma f[1 - G(pf)] \). Hence, we showed that a fine \( f < \hat{f} \) cannot be optimal for any value of \( v \). In summary, we showed that, in the parameter space \( v < \nu^nc \), the optimal fine \( f_{nc} \) is \( f_{nc} \in (v/[\sigma(1 - G(\frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)}])], \hat{f} \), and the optimal expected penalty is \( \hat{x}_{nc} = \frac{h}{1+\alpha-\sigma} - \frac{\sigma-\alpha}{\lambda(1+\alpha-\sigma)} \).
7.0.2 Proof of proposition 4 with full optimal policy set

Part a. This part follows the proof in 6.2.

Part c. We have shown in 6.2 that the solution is \( \hat{x}_c = p_c b_c(i, f) = h \). As in the clean regime, when \( \{ \text{PC}_c \} \) is slack \( p_c, f_c \) and \( i_c \) cannot be separately identified. Specifically, for any \( b_c(i, f) \in (v/\sigma(1 - G(h)), \sigma f) \), the optimal number of officers is \( p_c \) such that \( p_c b_c(i, f) = h \). We have left to prove whether or not it is optimal to select a bribe \( b_c(i, f) \leq v/\sigma(1 - G(h)) \), and, if so, what is the optimal expected penalty \( \hat{x}_c \). We will study this in the next section.

Part b. The solution is the same as in 6.2.

We have left to characterize the set of optimal policies for the parameter space \( v < v^c_0 \). Define \( \hat{b} \equiv v/\sigma(1 - G(h)) \). Depending on the choice of \( i, f \), the bribe can be larger than, equal to, or smaller than \( \hat{b} \). We will show that the optimal set has \( b \geq \hat{b} \).

Consider first what happens if the bribe chosen is \( b(i, f) = \hat{b} \). Then, both \( \{ \text{PC}_{nc} \} \) and \( \{ \text{LL} \} \) bind, and the correct solution (from part b) is given by \( \hat{x}_c = G^{-1}(1 - \frac{v}{\sigma f}) = h \). Thus, the welfare achieved is the same as in the case where \( b(i, f) > \hat{b} \) (see Part c.) Thus, the fine \( b(i, f) = \hat{b} \) is part of the optimal set.

Now, take any \( b(i, f) < \hat{b} \): For values of \( v \) such that \( v/\sigma[1 - G(\frac{b}{2})] \leq b(i, f) < \hat{b} \), part c applies; thus, \( \{ \text{PC}_{nc} \} \) and \( \{ \text{LL} \} \) both bind. The solution is \( \hat{x}_c = G^{-1}(1 - \frac{v}{\sigma f}) < h \). Thus, the bribe \( b(i, f) \) is not optimal.

For values of \( v \) such that \( b(i, f) < v/\sigma[1 - G(\frac{b}{2})] \), part a applies; thus, only \( \{ \text{PC}_{nc} \} \) binds. Hence, \( v > b(i, f)[1 - G(b(i, f)p')] \), where \( p' \) is the optimal number of officers for a given bribe \( b(i, f) \) and \( v \) in the interval under consideration. We construct a counter example bribe system that achieves a higher welfare. Specifically, choose a \( b'' \) larger than \( b(i, f) \) and such that \( b'' \in (v/\sigma (1 - G(h)), \sigma f) \) and a corresponding \( p'' \) such that \( 0 < p'' < p' \) and \( p''b'' = p'b(i, f) \). That is, \( b'' \) is in case c. Compare the welfare \( W_c(p', b(i, f)) \) and \( W_c(p'', b'') \):

\[
W_c(p', b(i, f)) - W_c(p'', b'') = -p'v + p'b(i, f)[1 - G(b(i, f)p')] < 0
\]

where the last inequality comes from noting that in the space under consideration \( v > b(i, f)[1 - G(b(i, f)p')] \). Hence, we showed that a bribe \( b(i, f) < \hat{b} \) cannot be optimal for any value of \( v \). In summary, we showed that, in the parameter space \( v < v^c_0 \), the optimal bribe \( b_c(i, f) \) is \( b_c(i, f) \in [v/\sigma (1 - G(h)), \sigma f] \), and the optimal expected penalty is \( \hat{x}_c = h \).

7.0.3 Mathematical proof of over- and under-enforcement in the clean regime in case b and c.

Case b The claim is that, when \( \{ \text{LL} \} \) and \( \{ \text{PC}_{nc} \} \) both bind (alternatively, for \( \nu_{nc}^\sigma \leq v \leq \tilde{v}_{nc}^\sigma \)), there is over-enforcement in the clean regime relative to the benchmark. Note that the optimal enforcement level in case (b) solves \( \sigma f[1 - G(p_{nc}f)] = v \). Substituting this into (4) yields \( pf = \frac{b}{1 + \frac{\sigma}{\sigma - \alpha}} = \frac{b}{1 + \frac{\sigma}{\sigma - \alpha}} \), which is optimal in the benchmark regime (i.e., \( p = p^*f \)) only when \( v = \tilde{v}_{nc} \). To complete the proof, then, we need to show that as \( v \) declines below \( \tilde{v}_{nc} \), enforcement increases more in the clean regime case than in the benchmark case. We use total differentiation of \( \hat{x} \) under the two regimes to compute \( \frac{\partial p}{\partial v} \):

\[
\frac{\partial p_{nc}}{\partial v} = \frac{-1}{f g(pf) \sigma f}
\]

23 What follows is the same proof that is used to prove proposition 6, part ii).
and
\[
\frac{\partial p^*}{\partial v} = -\frac{1}{fg(pf)} \left(1 + \alpha\right) + \frac{v}{1 - G(pf)}
\]
where, for the latter derivative, we made use of the fact that 
\[
\frac{v}{1 - G(pf)} = \lambda f[1 - G(pf)]
\]
whose derivative with respect to \( p \) is \( \frac{v}{1 - G(pf)} \). We thus need to prove that the former derivative is smaller than the latter in the region \( v_{\sigma}^{nc} \leq v \leq v_{\sigma}^{nc} \). This is so provided that \( \sigma f < f(1 + \alpha) + \frac{v}{1 - G(pf)} \); rearranging, we have the condition \( v > -(1 + \alpha - \sigma)f[1 - G(pf)] \), which is always the case because the right hand side is a negative number. QED.

**Case c** The claim is that, when \( \text{[PC}_{nc} \) is slack (alternatively, for \( v < v_{\sigma}^{nc} \)), there is an area of over-enforcement and an area of under-enforcement in the clean regime relative to the benchmark. In particular, for \( v \) that are close to \( v_{\sigma}^{nc} \), the clean regime over-enforces, and for \( v \) that are significantly lower than \( v_{\sigma}^{nc} \), the clear regime under-enforces.

Note from proposition 2 that enforcement in case c. is given by \( \hat{x}_{nc} = \frac{h}{\alpha - \sigma} - \frac{\sigma - \alpha}{\lambda(1 + \alpha - \sigma)} \). We show that this enforcement level is achievable by the honest benchmark case at a level \( v = \hat{v} < v_{\sigma}^{nc} \). Since enforcement in the honest regime increases as \( v \) decreases, it must be the case that for there is too much enforcement for \( v > \hat{v} \) and too little enforcement for \( v < \hat{v} \). We start by plugging \( \hat{x}_{nc} \) in the FOC for the benchmark case, equation (4). We have that
\[
\frac{h(1 + \alpha)}{1 + \alpha - \sigma} - \frac{(\sigma - \alpha)(1 + \alpha)}{\lambda(1 + \alpha - \sigma)} = h - \frac{v}{fg(\hat{x}_{nc})} + \alpha/\lambda
\]
Rearrange and simplify to obtain
\[
\hat{v} = \frac{\sigma f[1 - G(\hat{x}_{nc})]}{1 + \alpha - \sigma}(1 - \lambda h)
\]
Note that \( \hat{v} < \sigma f[1 - G(\hat{x}_{nc})] = v_{\sigma}^{nc} \) because of our assumption \( h > (\sigma - \alpha)/\lambda \), which excludes the corner solution \( p_{nc} = 0 \).

**7.0.4** Mathematical proof of over- and under-enforcement in the corrupt regime in case **c**.

We prove that, in the space \( v < \alpha f[1 - G(h)] \), the benchmark regime enforces more that the corrupt regime only when \( v < \alpha f[1 - G(h)](1 - \lambda h) \). We show that when \( \text{[PC}_{nc} \) is slack (alternatively, for \( v < v_{\sigma}^{nc} \)), there is an area of over-enforcement and an area of under-enforcement in the corrupt regime relative to the benchmark regime. Corrupt enforcement is given by \( \hat{x}_c = h \). The proof is based on showing that this enforcement level is optimal under the benchmark regime at a level \( v = \hat{v} < \alpha f[1 - G(h)] \). Since enforcement in the benchmark regime increases as \( v \) decreases, it must be the case that under the benchmark regime there is less enforcement for \( v > \hat{v} \) and more enforcement for \( v < \hat{v} \). We plug \( \hat{x}_c = h \) in the FOC for the benchmark regime, equation (4), and solving for \( \hat{v} \):
\[
\hat{v} = \alpha f[1 - G(h)](1 - \lambda h) < \alpha f[1 - G(h)]
\]

**8 Appendix C: Simulation Details (not intended for publication)**

Given that some of our solutions are not closed-form, we simulated and solved the model numerically using Matlab. The codes are available from the authors. We set \( f = 10.1 \), \( v \in (0, 4) \), \( \alpha = 0.5 \), \( h = 0.7 \) (low harm act) or \( h = 1.3 \) (high harm act), and \( \lambda = 1.2 \). We used an exponential
distribution function with support $[0, \infty)$. We looked at values of $\sigma \in (h/f, 1)$, which ensures that Assumption 1-3 are satisfied and the optimal number of officers $p$ is interior. In order to solve the model numerically we discretized $v$, $\sigma$, and $p$, using respectively 40, 13 and, 10003 grid points. Increasing the number of grid points does not change our results materially. To create the 2D graphs where $\sigma^*$ is depicted, we interpolated the welfare function in each regime to increase precision. Given the smoothness, continuity and monotonicity of our welfare functions in the regions under consideration, this was a harmless procedure.
Figure 1: The cost of corruption in clean (top) and corrupt (bottom) regimes for number of officers $p = 10\%$

Figure 2: Optimal welfare levels, clean and corrupt regimes
Figure 3: Optimal regime choice

Figure 4: Optimal welfare levels and regime choice with legalization
Figure 5: Regime choice with legalization, high harm activity

Figure 6: Regime choice with legalization, low harm activity
Figure 7: Enforcement level for a high harm activity, clean regime against tax-and-legalize

Figure 8: Enforcement level for a high harm activity, corrupt regime against tax-and-legalize
Figure 9: Enforcement level for a low harm activity, clean regime against tax-and-legalize

Figure 10: Enforcement level for a low harm activity, corrupt regime against tax-and-legalize