Addendum to Legalize, Tax, and Deter: Optimal Enforcement Policies for Corruptible Officials

Alfredo Burlando,[*] Alberto Motta [†]

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1 Legalization and development: proofs

In this section, we provide proofs for the statements made in section 5.2 of the paper: if the overall process of development increases parameter values x, h, v, f and decreases σ, the scope for legalization decreases. In section 3, we assume that development scales all parameters by a common factor β. In Section 3.1, we assume that the sensibility to corruption σ remains unchanged by development. Finally, in Section 4 we assume that x, h do not change. In all cases, we demonstrate that the scope for legalization (where the model predicts society is *strictly* better off due to legalization) decreases with development. In most cases, this simply means that the parameter $\tilde{z}$ decreases with development.

2 Key Equations.

$$W_{honest}(p, w, i, f) = \int_{pf}^{\infty} (x - h)g(x)dx - d\{pw - pf[1 - G(pf)]\}. \tag{1}$$

$$d\{v - \bar{f}[1 - G(p^* \bar{f})]\} = [h - p^* \bar{f}(1 + d)]g(p^* \bar{f})\bar{f}. \tag{2}$$

$$W_j(p_j, w_j, i_j, f_j) = \int_{\hat{x}_j}^{\infty} (x - h)g(x)dx - dp_j\{w_j - \mathbb{1}_{j=nc}(f_j - i_j)[1 - G(p_j f_j)]\}. \tag{3}$$

$$i \geq \sigma f \equiv i^{nc}. \tag{IC}$$

$$w_j + \Pr(audit\ success)_j \max[b_j(i, f), i_j] \geq v \tag{PC}$$

[*]University of Oregon, Department of Economics, Eugene, OR 97405, tel (541) 346-1351, e-mail: burlando@uregon.edu

[†]University of New South Wales, School of Economics, Australian School of Business, Sydney 2052, email: motta@unsw.edu.au

$$w_j \geq -\tilde{w} \equiv 0. \tag{LL}$$

3 Proof of proposition 1 (with scale parameter β)

We derive the optimal policy under prohibition here and subsequently compare optimal enforcement against the benchmark in the following subsection. Here we introduce the scale parameter β. We assume that more developed countries have higher market wages v, higher maximal fines $\bar{f}$, lower institutional tolerance for corruption σ, but also a higher return from the illegal activity x and a higher external cost h. Suppose that a country's set of parameters scales up at the same rate, $\{\beta v, \beta\bar{f}, \beta x, \beta h, \frac{\sigma}{\beta},\}$ where $\beta > 1$ is a measure of economic and social development.

Keep in mind that we have rescaled our parameters. A citizen that commits the illegal act faces an expected sanction pf. The benefit from the activity is βx. Hence, the citizen will commit the act if $\beta x \geq pf$ or $x \geq \frac{pf}{\beta}$. The new threshold is then $\hat{x}_{nc} = \frac{pf}{\beta}$ and the total quantity of crime is $1 - G(\frac{pf}{\beta})$, which is also the probability of a successful audit.

We are going to show that a higher β is associated with lower $\tilde{z}$ and a lower likelihood of adopting a tax-and-legalize scheme.

Case a. Consider first the case where (LL) is slack. Then, (PC_{nc}) binds and, from (PC_{nc}), we get that the optimal wage is $w_{nc} = \beta v - i[1 - G(\frac{pf}{\beta})]$. Replacing this in the welfare function (3), we get that

$$W^{\beta}_{nc}(p, f) = \int_{\frac{pf}{\beta}}^{\infty} \beta(x - h)g(x)dx - dp\{\beta v - f[1 - G(\frac{pf}{\beta})]\}. \tag{4}$$

Note first that welfare does not depend on i, and any $i \geq \frac{\sigma}{\beta}f$ satisfies (IC). In addition, by standard reasoning fines are maximal: $f = \beta\bar{f}$. Differentiating with respect to p, we find that the optimal interior solution p_{nc} is determined by the first order condition in the benchmark, honest case, equation (2).

We next determine the optimal i^*. As long as $\beta v > i^*[1 - G(\frac{p_{nc}\beta\bar{f}}{\beta})]$, (LL) is slack. Thus, any $i^* \in [\frac{\sigma}{\beta}\beta\bar{f}, \frac{v}{1-G(\frac{p_{nc}\beta\bar{f}}{\beta})}]$ ensures that (LL) does not bind. In particular, $i^* = \sigma\bar{f}$ is always a solution.

Next, we show the parameter space where this solution is feasible. The largest value of v such that (LL) binds is $\beta v = i^*[1 - G(p_{nc}\bar{f})]$; substituting this in (2) and considering that the hazard rate $\frac{[1-G(p_{nc}\bar{f})]}{g(p_{nc}\bar{f})} = \frac{1}{\lambda}$, we get

$$h - (1 + d)p_{nc}\bar{f} = \frac{di^*}{\beta\bar{f}\lambda} - \frac{d}{\lambda}. \tag{5}$$

Solving for $p_{nc}\bar{f}$ we get $p_{nc}\bar{f} = \frac{h}{1+d} - \frac{d(i^*/\beta\bar{f}-1)}{\lambda(1+d)}$. Substituting $p_{nc}\bar{f}$ and $i^* = \sigma\bar{f}$ into the participation constraint, we find that this solution is feasible for $\beta v > \tilde{z} \equiv \sigma\bar{f}\left[1 - G\left(\frac{h}{1+d} + \frac{d(1-\frac{\sigma}{\beta})}{\lambda(1+d)}\right)\right]$.

It remains to be demonstrated that, in the parameter space where the solution is feasible, it is also optimal. This follows immediately from the fact that when $v > \tilde{z}$, the government achieves benchmark welfare, and the benchmark welfare is strictly better than any clean regime welfare where (LL) binds. Thus, it is never optimal to set $i > \sigma\bar{f}$ in such a way that (LL) binds.[1]

[1] Assumptions A1-A2 ensure no corner solutions. $p_{nc} > 0$ is guaranteed if $v < \bar{f}(h\lambda+1)/d$ (always satisfied by assumption A2) and by assumption A1, which implies that $h > d(1 - \sigma)/\lambda$. $p_{nc} < 1$ is guaranteed by $v > \bar{f}g(\bar{f})\left(h - \bar{f}[1 + d] + 1/\lambda\right) = \lambda[h - (1 + d)\bar{f} + 1/\lambda]\bar{f}[1 - G(\bar{f})]$, which is always satisfied because the threshold is smaller than $\tilde{z}$ and by assumption A1.

Clearly a higher β is associated with lower $\tilde{z}$ and it makes this case (part a.) more likely to occur — a case where legalization is not bringing any particular improvement to welfare.

3.1 Proof of proposition 1 (σ is not scaled)

We derive the optimal policy under prohibition here and subsequently compare optimal enforcement against the benchmark in the following subsection. Here we introduce the scale parameter β. We assume that more developed countries have higher market wages v, higher maximal fines $\bar{f}$, but also a higher return from the illegal activity x and a higher external cost h. Suppose that a country's set of parameters scales up at the same rate, $\{\beta v, \beta\bar{f}, \beta x, \beta h\}$ where $\beta > 1$ is a measure of economic and social development. Here, we do not scale σ: tolerance of corruption is not improved by development.

Keep in mind that we have rescaled our parameters. A citizen that commits the illegal act faces an expected sanction pf. The benefit from the activity is βx. Hence, the citizen will commit the act if $\beta x \geq pf$ or $x \geq \frac{pf}{\beta}$. The new threshold is then $\hat{x}_{nc} = \frac{pf}{\beta}$ and the total quantity of crime is $1 - G(\frac{pf}{\beta})$, which is also the probability of a successful audit.

We are going to show that a higher β is associated with a lower likelihood of adopting a tax-and-legalize scheme.

Part a. Consider first the case where (LL) is slack. Then, (PC$_{nc}$) binds and, from (PC$_{nc}$), we get that the optimal wage is $w_{nc} = \beta v - i[1 - G(\frac{pf}{\beta})]$. Replacing this in the welfare function (3), we get that

$$W^{\beta}_{nc}(p, f) = \int_{\frac{pf}{\beta}}^{\infty} \beta(x - h)g(x)dx - dp\{\beta v - f[1 - G(\frac{pf}{\beta})]\}. \tag{6}$$

Note first that welfare does not depend on i, and any $i \geq \frac{\sigma}{\beta}f$ satisfies (IC). In addition, by standard reasoning fines are maximal: $f = \beta\bar{f}$. Differentiating with respect to p, we find that the optimal interior solution p_{nc} is determined by the first order condition in the benchmark, honest case, equation (2). It follows that $\hat{x}_{nc} = \hat{x}_{honest}$ and $W^*_{nc} = W^*_{honest}$.

We next determine the optimal i^*. As long as $\beta v > i^*[1 - G(\frac{p_{nc}\beta\bar{f}}{\beta})]$, (LL) is slack. Thus, any $i^* \in [\sigma\beta\bar{f}, \frac{v}{1-G(\frac{p_{nc}\beta\bar{f}}{\beta})}]$ ensures that (LL) does not bind. In particular, $i^* = \sigma\beta\bar{f}$ is always a solution.

Next, we show the parameter space where this solution is feasible. The largest value of v such that (LL) binds is $\beta v = i^*[1 - G(p_{nc}\bar{f})]$; substituting this in (2) and considering that the hazard rate $\frac{[1-G(p_{nc}\bar{f})]}{g(p_{nc}\bar{f})} = \frac{1}{\lambda}$, we get

$$h - (1 + d)p_{nc}\bar{f} = \frac{di^*}{\beta\bar{f}\lambda} - \frac{d}{\lambda}. \tag{7}$$

Solving for $p_{nc}\bar{f}$ we get $p_{nc}\bar{f} = \frac{h}{1+d} - \frac{d(i^*/\beta\bar{f}-1)}{\lambda(1+d)}$. Substituting $p_{nc}\bar{f}$ and $i^* = \sigma\beta\bar{f}$ into the participation constraint, we find that this solution is feasible for $\beta v > \tilde{z} \equiv \sigma\bar{f}\left[1 - G\left(\frac{h}{1+d} + \frac{d(1-\sigma)}{\lambda(1+d)}\right)\right]$. Thus, $\tilde{z}$ does not change with development (i.e., it is not affected by β); however, legalization still is less likely to occur as development occurs because it is more likely that, for a starting value of v, $\beta v > \tilde{z}$. The rest of the proof is similar to the one above.

4 Proof of proposition 1 (h and x are not scaled)

We derive the optimal policy under prohibition here and subsequently compare optimal enforcement against the benchmark in the following subsection. Here we introduce the scale parameter β. We assume that more developed countries have higher market wages v, higher maximal fines $\bar{f}$ and

lower institutional tolerance for corruption σ. Suppose that a country's set of parameters scales up at the same rate, $\{\beta v, \beta\bar{f}, \frac{\sigma}{\beta},\}$ where $\beta > 1$ is a measure of economic and social development.

Unilke in our previous proofs, here h and x are no longer rescaled. A citizen that commits the illegal act faces an expected sanction pf. The benefit from the activity is x. Hence, the citizen will commit the act if $x \geq pf$. The new threshold is then $\hat{x}_{nc} = pf$ and the total quantity of crime is $1 - G(pf)$, which is also the probability of a successful audit.

We are going to show that a higher β is associated with lower $\tilde{z}$ and a lower likelihood of adopting a tax-and-legalize scheme.

Part a. Consider first the case where (LL) is slack. Then, (PC_{nc}) binds and, from (PC_{nc}), we get that the optimal wage is $w_{nc} = \beta v - i[1 - G(pf)]$. Replacing this in the welfare function (3), we get that

$$W_{nc}(p, f) = \int_{pf}^{\infty} (x - h)g(x)dx - dp\{\beta v - f[1 - G(pf)]\}. \tag{8}$$

Note first that welfare does not depend on i, and any $i \geq \frac{\sigma}{\beta} f$ satisfies (IC). In addition, the welfare is equivalent to (1), the benchmark case without the possibility of corruption. The optimal policy thus follows the benchmark case, which means that $f = \beta\bar{f}$ and the optimal interior solution p_{nc} is determined by the following modified first order condition:

$$d\{v - \bar{f}[1 - G(p_{nc}\beta\bar{f})]\} = [h - p_{nc}\beta\bar{f}(1 + d)]g(p_{nc}\beta\bar{f})\bar{f}. \tag{9}$$

We next determine the optimal i^*. As long as $\beta v > i^*[1 - G(p_{nc}\beta\bar{f})]$, (LL) is slack. Thus, any $i^* \in [\frac{\sigma}{\beta}\beta\bar{f}, \frac{v}{1-G(p_{nc}\beta\bar{f})}]$ ensures that (LL) does not bind. In particular, $i^* = \sigma\bar{f}$ is always a solution.

Next, we show the parameter space where this solution is feasible. The largest value of v such that (LL) binds is $v = i^*[1 - G(p_{nc}\beta\bar{f})]$; substituting this in (9) and considering that the hazard rate $\frac{[1-G(p_{nc}\beta\bar{f})]}{g(p_{nc}\beta\bar{f})} = \frac{1}{\lambda}$, we get

$$h - (1 + d)p_{nc}\beta\bar{f} = \frac{di^*}{\beta\bar{f}\lambda} - \frac{d}{\lambda}. \tag{10}$$

Solving for $\hat{x}_{nc} = p_{nc}\beta\bar{f}$ we get $p_{nc}\beta\bar{f} = \frac{h}{1+d} - \frac{d(i^*-\beta\bar{f})}{\beta\bar{f}\lambda(1+d)}$. Substituting $p_{nc}\beta\bar{f}$ and $i^* = \sigma\bar{f}$ into the participation constraint, we find that this solution is feasible for $\beta v > \tilde{z} \equiv \sigma\bar{f}\left[1 - G\left(\frac{h}{1+d} + \frac{d(1-\frac{\sigma}{\beta})}{\lambda(1+d)}\right)\right]$. Clearly, this threshold is the same as the one derived in the first section (where x and h were scaled by β). The main conclusions from the proof in section 3 apply.