

# Individual Subjective Discounting: Form, Context, Format, and Noise

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## ABSTRACT

Debate about the appropriate value for any single social discount rate for public projects stems in part from our lack of knowledge about how individual discount rates vary across people and across choice contexts. It would be valuable to be able to estimate aggregate willingness to pay for a public project as a function of heterogeneous individual discount rates. We could then contemplate a strategy of counterfactual simulation of aggregate willingness to pay for the same project under the systematically lower discount rates that are often argued to be compatible with the normative goal of intergenerational equity. Using a sample of roughly 15,000 choices by over 2000 individuals, we estimate prototype utility-theoretic models concerning private tradeoffs involving money over time that reveal individual-specific discount rates. Statistically significant heterogeneity in discount rates is quantified along many dimensions for an exponential discounting model, a competing hyperbolic model, and a generalized hyperbolic model. We control for experimentally differentiated choice scenarios and elicitation formats, for sociodemographic heterogeneity, and for complex forms of heteroscedasticity.

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# Individual Subjective Discounting: Form, Context, Format, and Noise

Whenever the benefits and costs of a non-tradable durable good or a public good have different time profiles, discounting is a necessary step in any assessment of that good's desirability. A pervasive feature of the existing social choice literature is the notion that we need one common discount rate for social decision-making and that the main challenge surrounds our choice of which single discount rate to apply (e.g. Moore et al. 2004). One single discount rate is a convenient assumption for many models. If capital markets were perfect, and if all goods were tradable, some would argue that the market interest rate should accurately reflect everyone's intertemporal preferences. However, factors such as transactions costs, the incompleteness of intertemporal markets, and people's tendencies to define artificially separate budgets for different activities invalidate the assumption of perfect capital markets. This foils any expectation that individuals will adjust levels of present and future consumption to bring marginal rates of substitution into line with a single market interest rate.

Economists generally have a strong bias toward revealed preferences, so the profession customarily looks for evidence of society's average tradeoffs between present and future consumption in markets where these preferences may be exercised by individuals, and where their combined behavior will result in an aggregate measure of discount rates. In recent years, however, economists have become more willing to consider direct survey methods and respondents' stated behavior as a potentially valuable source of information about preferences. For example, Manski (2004) describes in detail how economists have begun to rely increasingly on survey data regarding individuals' probabilistic expectations about significant personal events. Cross-sectional differences in these expectations are crucial to many economic models, but are difficult to infer at the individual level from revealed preference

analyses. The same is true of individual discount rates. Stated preferences, like those employed in the present study, can be a potentially valuable source of information, an insight that has also been recognized in Shapiro (2005).

Both Lind (1990) and Arrow et al. (1996) argue that discount rates should be based on how individuals trade off between present and future consumption, and that these rates are indeed likely to differ contextually. At the individual level, at least for non-tradable goods, the discount rate is an artifact of preferences for current versus future consumption, just as willingness to pay for different commodities is an artifact of preferences for the contemporaneous consumption of different goods and income. There is no reason why marginal rates of time preference should be any less individual, or less context-specific, than marginal rates of substitution between contemporaneous goods.<sup>2</sup>

The quest for a single representative discount rate to use in making social choices is in part a *normative* issue. Given that future generations are inadequately represented in current social decision-making, we often elect to override the discount rates that are implicit in the unconstrained choices of this current generation. We often wish to know, instead, “What would be the choices of this population if discount rates were lower?” To be able to simulate these counterfactual choices, we need to conduct a thorough *positive* analysis of why individual choices made by members of the current generation are as they are. However, the extent of heterogeneity in the relevant individual discount rates tends to be largely unquantified. Only a few large-scale and comprehensive examples of empirical individual discount rates are presently available, notably Warner and Pleeter (2001) and Harrison et al. (2002). However, neither of these studies spans all of the dimensions that can be of interest in different contexts.

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<sup>2</sup>The individual discount rate is equal to the marginal rate of time preference, minus one. The “pure” rate of time preference differs from the individual discount rate in that it is evaluated along the individual’s intertemporal budget constraint at a point corresponding to equal amounts of current and future consumption (Lawrance, 1991).

Consider how our limited understanding of individual discount rate heterogeneity impairs social choices. For individual  $i$ , where  $i = 1, \dots, N$ , let  $b_{it}$  represent *net* benefits in periods  $t = 1, \dots, T$  and let  $(b_{i1}, b_{i2}, \dots, b_{iT})$  be the time profile of net benefits.<sup>3</sup> In the absence of information about individual-specific time-preferences, the present discounted value of aggregate net benefits of a particular durable or public good in each future period,  $\sum_{i=1}^N b_{it}$ , must be computed using some aggregate discount rate,  $r_a$ :

$$PDV_a = \sum_{t=1}^T (1 + r_a)^{-t} \left( \sum_{i=1}^N b_{it} \right) \quad (1)$$

In contrast, a formula that honors individual time preferences would use individual discount rates,  $r_i$  :

$$PDV_i = \sum_{i=1}^N \left( \sum_{t=1}^T (1 + r_i)^{-t} b_{it} \right) \quad (2)$$

In this case, the first step is to discount individual net benefits back to the present using a discount factor appropriate for that individual,  $(1 + r_i)^{-t}$ . The second step is to aggregate these individual discounted net benefits into a measure of social benefits. The practical problem for implementing this alternative measure is that we typically do not know much about the values of  $r_i$ , for an individual with particular attributes, that might apply in a particular choice context.

In this paper, we propose and demonstrate a strategy for the measurement of individual-specific discount rates via survey methods. We first lay out a formal random utility framework to accommodate the conceptual problem of consumer choice when the individual faces a time profile of costs in order to obtain some time profile of services of a durable or public good, as well as a second choice concerning the receipt of money with different time profiles. This general model involves two types of choices because we envision that this approach will have

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<sup>3</sup>We will not discriminate between benefits in terms of consumption and benefits in terms of utility.

value in producing improved estimates of fitted individual discount rates that can be used simultaneously to explain variation in other individual decisions in a multi-equation system. For identification, as in any simultaneous equations model, it will be important to include exogenous determinants of individual discount rates that do not also explain the durable or public goods choice.

For our empirical example, we focus on just the individual discount rate portion of the larger model. We query individuals concerning their preferred way to receive lottery winnings—either as a stream of annual payments over some time horizon, or as a smaller lump sum in the current period. With these choice data, we can generate empirical estimates of exponential and simple hyperbolic discount functions and also describe the results of attempting to fit a more general hyperbolic form. Our models accommodate broad heterogeneity in preferences and complex forms of heteroscedasticity in the underlying indirect utility specification. We also assess the sensitivity of estimated discount rates and error variances to a number of alternative choice scenarios and elicitation formats.

The main idea we wish to promulgate, by framing our model in this way, is that human behavior with respect to related choices should be consistent. One utility function should underlie all of the choices made by any single individual. Any random utility model we use to take advantage of choices that highlight subjects' individual discount rates should also be able to accommodate these same individuals' choices with respect to durable or public goods. The random utility models used to capture each type of choice will have common preference parameters, so the different types of choices can be pooled and estimated jointly. The specific discount rate choice and the additional durable or public goods choice can be combined in one model to improve our chances of identifying a wider range of preference parameters. A discount rate is an attribute of individual preferences that is not usually separately identifiable within the context of a single durable or public goods choice unless some strong assumptions are made. However, if durable or public goods choice data can be

combined with other choices by the same individuals that expressly and exclusively involve tradeoffs of money over time, within a fully compatible utility specification, then there is some hope for separately identifying heterogeneity in discount rates and heterogeneity in preferences for durable or public goods.

## I. Background

### *A. Contextual Differences in Empirical Discount Rates*

Over the last twenty years or so, both economists and psychologists have explored factors that can affect individual discount rates. Frederick et al. (2002) provide a thorough survey of theoretical and empirical research concerning time discounting and time preference. They tabulate over forty attempts at empirical estimation of discount rates according to (a) type (experimental or field), (b) good(s) (money, life-years, etc.), (c) real or hypothetical, (d) elicitation method (choice, matching, rating, or pricing), (e) time range (from less than one day, to 57 years), and finally, (f) the range of implied discount rates and their associated discount factors. What is striking about this summary is the extraordinary variance, across studies, in empirically estimated private discount rates across different choice contexts (even without considering possible systematic differences across sociodemographic groups).

This accumulating evidence about context effects strengthens the case for departing from the convention of using one representative discount rate in decision-making. Where there are substantial groupwise differences in discount rates, it may be very important to preserve these differences in net benefits estimation. It is also possible that differences in discount rates across contexts (long- or short-term tradeoffs, private or public tradeoffs) will be sufficiently large that just one menu of group-specific discount rates will be insufficient. All of this points to a need for new techniques to elicit reliable group- and context-specific discount rates. We describe one possible technique.

### *B. Discounted Utility Anomalies and Hyperbolic Discounting*

Frederick et al. (2002) identify economists’ reliance on the expedient single-parameter discounted utility (DU) model suggested by Samuelson (1937) as one of the impediments to progress in discount rate research. They inventory the suite of DU-anomalous results that have induced a number of researchers to think about other representations of discounting behavior. Many researchers have now explored these anomalies (e.g. Loewenstein and Thaler (1989), Loewenstein and Elster (1992), and Loewenstein and Prelec (1992)). Frederick et al. (2002) emphasize that individual intertemporal tradeoffs can reflect a whole host of different processes that play out at the individual level, not just “pure time preference.” Among possible confounding factors, they enumerate consumption reallocation, intertemporal arbitrage, concave utility, uncertainty, inflation considerations, expectations of changing utility, and the collection of tendencies labeled as habit formation, anticipatory utility, and visceral influences.

Among alternative discounting formulas, a generalized hyperbolic discount function,  $\phi_g(t)$ , is discussed by Loewenstein and Prelec (1992):

$$\phi_g(t) = (1 + \gamma t)^{-\beta/\gamma}, \quad \beta, \gamma > 0 \quad (3)$$

where the  $\gamma$  parameter dictates how far the function departs from constant (exponential) discounting. The  $\phi_g(t)$  form was defined by Harvey (1986), and derived axiomatically by Prelec (1989). For the empirical application in the present paper, it is relevant that as  $\gamma$  goes to zero, the generalized hyperbolic function becomes the standard continuous-time exponential discounting function,

$$\phi_e(t) = \exp(-\beta t) \quad (4)$$

Generalized hyperbolic functional forms seem to have had their genesis with a one-

parameter special case,  $\phi_m(t) = (1 + \gamma t)^{-1}$ , proposed by Herrnstein (1981) and explored further by Mazur (1987). The  $\phi_m(t)$  form involves the constraint  $\beta = \gamma$ . Harvey (1986) suggested an alternative hyperbolic case,

$$\phi_h(t) = (1 + t)^{-\beta} \tag{5}$$

embodying the constraint that  $\gamma = 1$ .

In the empirical section of this paper, we will employ three different functional forms in our models of discounting choices: the standard exponential model,  $\phi_e(t)$ , the simple hyperbolic model,  $\phi_h(t)$ , and the generalized hyperbolic model,  $\phi_g(t)$ , that has the exponential and simple hyperbolic models as special cases when  $\gamma = 0$  and  $\gamma = 1$ .

### *C. Digression: Weitzman's Expert Survey Sample*

Weitzman (2001) arrives at a functional form for discount rates identical to the generalized hyperbolic model in equation (3), but via a different route.<sup>4</sup> From a survey of the opinions of over 2,000 professional Ph.D.-level economists, he determines that the social discount rates advocated by these experts, measured in percentage points, range from  $-3$  to  $+27$ , with a sample mean of about 4 and a standard deviation of about 3. He notes that the empirical marginal distribution appears to compare favorably, in terms of its shape, to a gamma probability density function. The key empirical insight is that individual expert opinions about social discount rates vary rather substantially. Still, Weitzman's goal is to develop a model to produce a single social discount rate for policy evaluation that nevertheless accommodates heterogeneous opinions of experts concerning the intertemporal tradeoffs individuals should be willing to make.<sup>5</sup>

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<sup>4</sup>Weitzman does not, himself, identify the similarity between his specification and that of Loewenstein and Prelec (1992).

<sup>5</sup>For details concerning the relationship between Weitzman's approach and the Loewenstein and Prelec (1992) model, see Appendix I.



Weitzman acknowledges that “...the average panel member knows about, and typically does not feel acutely uncomfortable with, the approximation of constant exponential discounting. The primary disagreement among panel members is over the appropriate value of the as-if-constant discount rate.” (p. 264). Weitzman notes that the amount of “uncertainty” about discount rates in the sample generates a sliding-scale effective discount-rate schedule, whose decline over time is significant enough to recommend that it be incorporated into discounting of long-term projects.<sup>6</sup> We believe that this “uncertainty” might be better characterized as “heterogeneity”—systematic differences in preferences across individuals.

Had Weitzman collected the characteristics of each of his sample of economists, he might have fit a regression-type model to explain the differences in their subjective social discount rates. However, he only differentiates between 50 “leading” economists and other economists who answered his survey. His point estimate for the mean preferred social discount rate in the “leading” sample differs from that for the other group, being slightly larger, but he undertakes no formal hypothesis testing, nor does he seek to use any more than just this implicit dummy variable to look for systematic differences in discount rates. In this paper, we explore the generalized hyperbolic discounting model and its two special cases, using a wide variety of factors to accommodate systematic heterogeneity across our sample.

#### *D. Empirical Estimation of Individual Discount Rates*

A seminal paper by Hausman (1979) uses observed household choices among consumer durables (air conditioners) with higher and lower capital and operating costs to infer discount rates. In one model, these rates are allowed to vary with income levels.<sup>7</sup>

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<sup>6</sup>Newell and Pizer (2000) delve further into the consequences of uncertain discount rates. When the expectation is taken with respect to a density function for "the" discount rate, the expected discounted value of future benefits will be greater than the discounted value computed using the expected discount rate, since discounted values are a convex function of the discount rate.

<sup>7</sup>Gately (1980) uses a similar approach with refrigerators and finds even higher implied discount rates (although he does not report any systematic heterogeneity in discount rates).

An experimental setting with numerous “matching”-type questions per subject is used by Kirby and Maraković (1995). This strategy allows estimation of individual-specific  $\phi_e(t)$  and  $\phi_h(t)$  discount functions, but does not extend to explaining heterogeneity in terms of any observable individual characteristics. Cairns and van der Pol (1997) also ask matching questions (of roughly 500 survey subjects) concerning both “short run” and “long run” choices. They find evidence favoring the non-constant discounting models over the conventional constant discounting model.<sup>8</sup> The possibility of casting discount rates as systematic varying parameters is noted, but they indicate that their data were not collected to make these distinctions.

Survey “choice”-type questions (the family of methods employed in the present study) appear to have been first used to infer marginal rates of time preference by Johannesson and Johannsson (1997), albeit in a health context, and they do not explore how this rate itself varies with sociodemographics. Intertemporal preferences for health are also elicited via survey in van der Pol and Cairns (2001). Implied discount rates for two samples of about 400 respondents vary according to whether own health or others’ health is being considered, but these discount rates are not systematically differentiated by sociodemographics.

In another vein, Chesson and Viscusi (2000) address discounting jointly with uncertainty, estimating implicit rates of time preference with respect to deferred gambles. They find that estimated discount rates decrease with the time horizon of the gamble, a result that is consistent with the predictions of Loewenstein and Elster (1992) concerning time horizon effects. However, they acknowledge that “the combined tasks of discounting and probability assessment exceed the cognitive capabilities of many survey subjects.”

The only very large-sample empirical estimates of discount rates in a revealed preference context are offered by Warner and Pleeter (2001). These authors analyze the decisions of

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<sup>8</sup>They base their assessments on the sums of squared deviations between the actual empirical discount factors conveyed by respondents, and the fitted discount factors implied by each model, where these factors are determined by line-searches.

many thousands of US military personnel concerning a choice between a lump-sum separation benefit or an annuity, relying on a reduced-form model for the latent individual discount rate based on the discrete choices of individual subjects between their two payment alternatives.<sup>9</sup> Statistically significant heterogeneity in the implicit exponential discount rates is confirmed, but there is little formal attention paid in the paper to the distinction between exponential and hyperbolic discounting models. The huge cross-sectional samples also raise the usual questions about heteroscedasticity, but its presence in the model is not assessed.<sup>10</sup> In the present paper, we consider alternative discounting models and we explicitly model the variances of the errors, which are related to choice consistency across subgroups. (See Swait and Louviere, 1993; DeShazo and Fermo, 2002).

Harrison et al. (2002) describe a smaller field experiment using a representative sample of 268 people in Denmark between the ages of 19 and 75. Their decision-makers seem to employ nominal discount rates that are constant over one- to three-year horizons. They explore two main elicitation strategies: 109 observations are spread across four single-horizon contexts and 132 observations are allocated to multiple-horizon choice contexts.<sup>11</sup>

Two very recent examples should also be mentioned: Keller and Strazzera (2002) estimate both  $\phi_e(t)$  and  $\phi_h(t)$ , but use an existing data set from Thaler (1981) to generate simulated data for their analyses. They mention the possibility of, but do not pursue, systematically varying individual discount rates. Survey data concerning choices among alternative

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<sup>9</sup>Such models have actually been in wide use for many years in the environmental non-market valuation literature, since they are suitable for the analysis of referendum contingent valuation data. These models admit for structural choice modeling, but these authors do not pursue a formal random utility framework for their analysis.

<sup>10</sup>Warner and Pleeter are careful to control for self-selection into their observed sample from the overall population of military personnel (not all of whom were eligible for the payment choice). But they still cannot compensate for non-random selection into the military in the first place, which may render these samples very different from the general population in terms of unobservables. We still have no real idea whether the estimated discount rates for their two samples are any more representative of rates in the general population (of similar ages and education levels) than are discount rates estimated from other special samples of respondents.

<sup>11</sup>The resulting interval data for the implied individual discount rates is estimated using an algorithm in Stata 7 that accommodates error correlations within the set of choices by each participant.

climate change mitigation programs in the context of forest loss prevention are used by Layton and Levine (2002). In conjunction with Weitzman's social discount rate distribution for professional economists, they calculate posterior and prior odds in favor of several discount rate intervals. This approach accommodates heterogeneity in discount rates but does not parameterize the relationship between the sizes of discount rates and other individual characteristics.

## II. Formal Random Utility Choice Framework

Previous empirical estimates of discount rates have not been derived from utility-theoretic specifications. Most researchers employ ad hoc (reduced-form) specifications for stand-alone discount rate equations. Thus, these models for individual discount rates do not lend themselves to seamless integration with other models of related choice behavior by the same individuals. Our model is intended to accommodate, formally, two types of hypothetical choices by each individual. One is a choice concerning a non-tradable durable good or a long-lived public good. The second choice (upon which we will focus in this paper) is a choice concerning whether the individual would prefer to take lottery winnings as (a.) a series of annual payments, or (b.) a smaller lump sum now.<sup>12</sup>

In the most general specifications of the random utility model presumed to underlie both types of choices, we will assume that the individual's current utility  $v_i$  depends linearly upon their current net income,  $y_i$  (as a proxy for the consumption of all other goods), and their current flow of services from the durable or public good,  $g_i$ . In the case of homogeneous preferences, an individual's current utility might be expressed as:

$$v_i = \mu y_i + \delta g_i + u_i, \quad i = 1, \dots, n \quad (6)$$

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<sup>12</sup>Hypothetical lottery winnings scenarios have also been used by Thaler (1981). Thaler's hypothetical lottery, also using students, involved the simple choice between an early prize of \$15 (or \$75 or \$250 or \$3000), and later prizes paid in 3 months, 1 year, or 3 years. It produced some very large discount rates, which he speculates may be due to "the hypothetical nature of the questions and the youth of the subjects."

where  $u_i$  is an error term.<sup>13</sup>

Current utility is not the sole determinant of choices in cases where the individual faces different time profiles for future costs and benefits. Given the assumed linearity of the function, assume that the stream of future benefits from the flow of services of the durable or public good can be converted into a present discounted value  $G_i$ . For the money-denominated argument of utility, the durable good choice also implies a change in the present discounted value of future net income,  $Y_i$ . In the simplest example, net income can differ because of a one-time initial capital cost,  $C_i$ , and constant per-period operating cost of  $c_i$ .

In this exposition, we will use just a binary choice concerning whether to purchase one specific model of a durable good, or, analogously, to vote in favor of the provision of a particular long-lived public good.<sup>14</sup> In binary choice models,  $\Delta V_i$ , the difference in utility levels across the two alternatives in the stated choice scenario, is presumed to drive the individual's choice. In our linear model, this difference in discounted utilities will depend upon the difference in discounted net income levels,  $\Delta Y_i$ , and the difference in discounted net durable- or public-good benefits,  $\Delta G_i$ . For completeness, the discounted error term would also need to be distinguished:  $u_i^*$ .

For conventional exponential discounting with individual discount rate  $r_i$ , in discrete time, the exponential discount factor is  $\phi_e(t) = (1 + r_i)^{-t}$ . If earned income would be the same across the two choices in all relevant future periods up to  $T_i$ , its level would net out

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<sup>13</sup>Of course, a linear specification for utility is convenient (if supportable) because it allows one to ignore the usual problem of departures between the discounting of utility and the discounting of consumption. However, nothing precludes generalization to a nonlinear function. In other work-in-progress with a different data set, we entertain a constant relative risk aversion utility function, for example. The exposition is merely simpler in the linear case, so we use a linear example here.

<sup>14</sup>It would be straightforward to generalize our model to accommodate not just the choice of whether to buy a durable good, but also a choice between several durable goods. This could lead to a multinomial logit choice model for the durable good decision.

of the linear utility difference in the following formula for  $\Delta V_i$ :

$$\Delta V_i = \mu_i \left\{ \left[ y_i - C_i - c_i \sum_{t=0}^{T_i-1} (1+r_i)^{-t} \right] - [y_i] \right\} + \delta_i \Delta G_i + \varepsilon_i \quad (7)$$

where  $\varepsilon_i$  is the difference in the  $u_i^*$  error terms associated with discounted utility levels under the two alternatives.<sup>15</sup> As usual, the error dispersion parameter,  $\kappa_i$ , and the marginal utility parameters cannot be separately identified, so  $\kappa_i$  must be normalized to unity, at least for some baseline subgroup.

When such a durable- or public-goods choice is used alone, it is often difficult to separately identify all three of the parameters  $\mu_i/\kappa_i$ ,  $\delta_i/\kappa_i$ , and  $r_i$ . However, in our study, we can use the same basic utility difference function to model a second choice, concerning how to take lottery winnings, which involves no difference in the net present value of the services of a durable goods. Here,  $L_i$  is the optional current-period lump sum lottery disbursement, to be compared against a sequence of  $T_i$  annual payments in the amount  $p_i$ , starting today.

$$\Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ [y_i + L_i] - \left[ y_i + p_i \sum_{t=0}^{T_i-1} (1+r_i)^{-t} \right] \right\} + \frac{\delta_i}{\kappa_i} [0] + \frac{\varepsilon_i}{\kappa_i} \quad (8)$$

From this type of choice used alone, it is not possible to identify both  $\mu_i/\kappa_i$  and  $\delta_i/\kappa_i$ , but greater resolution can be obtained for  $r_i$ . The key insight is that pooling the types of choices in equations (7) and (8) may allow all parameters to be readily identified. If the same discount rate applies to both increases and decreases in future net income, the durable/public goods choice can serve to identify the marginal rates of substitution between money and the services of the durable/public good, whereas the lottery winnings choice can more clearly identify the individual discount rate.

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<sup>15</sup>The properties of the underlying current-period errors,  $u_i$ , may be assumed to be such that the transformed and differenced  $\varepsilon_i$  errors in any equation for  $\Delta V_i$  conform to the logistic distribution underlying an ordered logit model.

Concentrating on just the lottery winnings portion of the joint choice model, simple hyperbolic discounting (in the sense of Harvey (1986), using  $\phi_h(t)$ ) would lead to an alternative utility difference function:

$$\Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ [y_i + L_i] - \left[ y_i + p_i \sum_{t=0}^{T_i-1} (1+t)^{-\beta_i} \right] \right\} + \frac{\delta_i}{\kappa_i} [0] + \frac{\varepsilon_i}{\kappa_i} \quad (9)$$

where  $\beta_i$  is the hyperbolic discounting parameter. For generalized hyperbolic discounting,  $\Delta V_i$  for the lottery winnings choice will involve an additional parameter,  $\gamma$ , as part of the  $\phi_h(t)$  discount factor:

$$\Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ [y_i + L_i] - \left[ y_i + p_i \sum_{t=0}^{T_i-1} (1+\gamma t)^{-(\beta_i/\gamma)} \right] \right\} + \frac{\delta_i}{\kappa_i} [0] + \frac{\varepsilon_i}{\kappa_i} \quad (10)$$

The full specifications described in variants (8), (9), and (10) are appropriate models to handle the *joint* estimation of (a) a policy choice when the individual faces different costs at different times in order to obtain a change in the present value of a flow of services from public goods  $\Delta G_i$ , and (b) an auxiliary financial time-wise tradeoff such as our lottery winnings choice. Our empirical illustration, however, focuses on just the subsidiary problem of identifying individual-specific discount rates (either  $r_i$  or  $\beta_i$ , and possibly  $\gamma$ ), so some of the generality of the theoretical specification described above is not exploited here.<sup>16</sup>

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<sup>16</sup>In principle, it is possible to allow the underlying indirect utility function to be non-linear with respect to net income. If the magnitudes of these costs are sufficiently large to represent a substantial portion of the individual's income, it should be possible to separately identify individual-specific degrees of risk aversion (nonlinearity in income in the indirect utility function) as distinct from individual-specific discount rates. For our data, there appears to be insufficient variability in net incomes to permit a distinction between discounting and risk aversion if risk aversion in preferences is permitted. Also, since the stream of future payments and the current period lump sum that define our specific choice scenario are represented as being certain, it is not too surprising that there is insufficient information in the answers to the lottery choice question to accurately identify the individual's degree of risk aversion. Chesson and Viscusi (2000) also have difficulty introducing risk aversion as a property of preferences that is distinct from discounting. Their hypothetical choice scenarios compound uncertainty and discounting, however, and they end up estimating implicit discount rates with the acknowledgement that failing to allow for risk aversion will bias these estimates downward.

In our estimating data, only the present discounted value of net income differs across the alternatives posed in the stated preference choice scenario about lottery winnings. Thus  $\Delta G_i = 0$  for all choices. Furthermore, there is no information upon which to estimate  $\delta_i/\kappa_i$ , the normalized marginal indirect utility from the public good. The “difference” upon which the choice is based is essentially just a difference in net discounted income, with the dispersion parameter  $\kappa_i$  serving to scale the index function of the choice model so that the error distribution can be logistic. Our estimating specifications for the lottery winnings choice become:

$$\text{Exponential:} \quad \Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ L_i - p_i \sum_{t=0}^{T_i-1} (1 + r_i)^{-t} \right\} + \frac{\varepsilon_i}{\kappa_i} \quad (11)$$

$$\text{Simple hyperbolic:} \quad \Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ L_i - p_i \sum_{t=0}^{T_i-1} (1 + t)^{-\beta_i} \right\} + \frac{\varepsilon_i}{\kappa_i} \quad (12)$$

$$\text{Generalized hyperbolic:} \quad \Delta V_i = \frac{\mu_i}{\kappa_i} \left\{ L_i - p_i \sum_{t=0}^{T_i-1} (1 + \gamma t)^{-(\beta_i/\gamma)} \right\} + \frac{\varepsilon_i}{\kappa_i} \quad (13)$$

These measures of the indirect utility differences,  $\Delta V_i$ , are the building blocks for our random utility econometric models. Appendix II describes in detail the construction of an appropriate log-likelihood function for our data. Briefly, the log-likelihood function is a sum of component ordered-logit log-likelihoods for choices with two, three, four, and five possible responses pooled over independent samples. We impose non-negativity on the scalar marginal utility of income and on the systematically varying discount parameters by specifying them as  $MUI = \exp(\mu)$ ,  $r_i = \exp(r'Z_i^r)$  and  $\beta_i = \exp(\beta'Z_i^\beta)$ . The vectors of explanatory variables,  $Z_i^r$  and  $Z_i^\beta$ , need not be identical, but we will drop their superscripts.

There is also the matter of “noise”—the extent of heteroscedasticity in the error terms in the random utility specification. Utility-difference error variances have been argued to affect



“choice consistency” (DeShazo and Fermo, 2002). In any stated preference context, there is always a concern that the quality of the choice information elicited in a hypothetical choice scenario is dependent upon (a) how seriously the respondent takes the choice exercise, (b) how difficult they find the choice task, (c.) their prior experience in similar choice situations, and (d.) any constraints that prevent them from considering their choices sufficiently carefully. The dispersion parameter  $\kappa_i$  must always be normalized to unity for some base category of respondents, but it is often appropriate to allow the dispersion for other groups to differ systematically relative to this normalizing group. We accommodate very rich stochastic structures in our estimating specifications.

### III. Survey Sample

Our data are derived from an web-based (Internet) survey with over 2,000 participants from a wide variety of classes at universities throughout the United States and Canada. It can be viewed as a national and international extension of the typical “classroom survey,” but there is no pretense that this sample represents the U.S. and Canadian populations, or even the population of college students in these countries. There are significant disparities across institutions in access to web-based resources, across classes in the salience of the larger survey topic (global climate policy), and in the opportunity costs of students’ time spent in completing the survey.<sup>17</sup> The module of the survey that was designed specifically to elicit individual discount rates asks the respondent to imagine they have just won a lottery. They are asked to choose between taking their winnings as a series of  $T$  annual installments, starting “today,” or as a smaller overall lump sum payable immediately. Table 1 outlines our

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<sup>17</sup>For social choice problems involving very long time horizons, it can be argued that the preferences of today’s young people deserve particular attention, since they will be the surviving (net) beneficiaries of whatever policies are adopted in the near term. And while survey research is inevitably vulnerable to criticism based upon its hypotheticality, we at least pursue the issue of “construct validity” very aggressively in this paper. It is crucial, for example, to assess whether there is systematic variation in the error dispersions in one’s model and to determine whether the nature of this variation (which cannot be avoided) is plausible.

available variables and provides descriptive statistics for our sample. Figure 1 reproduces one variant of the survey page for these discounting choices.

Across respondents, the dynamically loaded survey page randomly varies the size of the annual installments among \$300, \$600, \$1200, \$2400, \$3600, and \$4800, and the number of these installments (the time horizon) among 20, 30, and 40 years.<sup>18</sup> The sizes of the annual installments were intended to reflect increased monthly costs of \$25, \$50, \$100, \$300, and \$400 for a public good, and the time horizon captures some of the more-expensive and longer-term environmental programs, such as climate change mitigation.<sup>19</sup> Each respondent is presented with an ordered list of lump sums and asked to indicate whether (and sometimes, to what extent) they would prefer each lump sum to the single pattern of annual installments. Coincidentally, this application spans the type of context relevant to the theoretical discussion in Karp (2005).

A selection of sociodemographic characteristics was elicited after the various choices in the survey had been recorded, including age brackets, gender, educational attainment, field of study, whether courses have been taken in economics, work status, political ideology, and family income bracket. Some less conventional variables were also collected. To proxy for individual capital market constraints, we ask for an estimate of the largest sum of money the individual believes they could qualify to borrow, without collateral. The survey software also keeps track of timing as respondents progress through the survey.<sup>20</sup>

Finally, preference elicitation formats were randomized across respondents. Some saw a long list of thirteen lump sums, some saw seven, five, or only three, although the range

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<sup>18</sup>Since the majority of the subjects are students, the 40-year time horizon is less likely to conflict with life expectancies than it would in a general population sample.

<sup>19</sup>In no case are the annual payments large enough to substitute for the subject's entire annual income from working. There is some anecdotal evidence suggesting that when the amounts become large enough to be comparable to labor income, the annual option becomes relatively more attractive than the lump sum, since it affords an option to quit one's job and still count on a steady stream of income (at least in nominal terms).

<sup>20</sup>The online survey was programmed in Perl by the authors.

in lump sums was identical across these four different designs. The objective exponential discount rates implicit in the list of lump sums presented to respondents who saw all thirteen lump sums were integer values between 1 percent and 10 percent, as well as 12 percent, 15 percent and 20 percent.<sup>21</sup> Across respondents, lump sums were either increasing or decreasing in size from the top to the bottom in the list. Finally, the format of the actual choice with respect to each lump sum involved different numbers of response options (just two, for “Yes/No”, up to five levels including “Definitely Yes”, “Probably Yes”, “Not Sure”, “Probably No”, and “Definitely No”). The horizontal ordering (Yes to No, No to Yes) of these answer options was also randomized. By using an assortment of elicitation formats, we can assess the effect of any one format selection on the discount rates that we infer from respondents’ choices. In evaluating the potential usefulness of a lottery-winnings question such as this, the robustness of the results to alternative elicitation formats is an important consideration.

#### **IV. Empirical Results**

To explain heterogeneity in discount rates in this study, there are three main types of explanatory variables in play: those that describe the randomly assigned context of the choice from which individual discount rates are inferred, those that measure individual-specific characteristics, and those that describe the particular variant of the randomized design of the elicitation format. Across all of the models in Table 2, the marginal utility of income parameter serves a role that is indistinguishable from that of the baseline error variance—it merely scales the magnitude of the systematic portion of the choice model so that the error variance conforms to the assumptions of a logit-based choice model. We will not comment further on this parameter.

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<sup>21</sup>Early versions of the survey included 30 percent and 50 percent implicit rates, but the very small corresponding lump sums were overwhelmingly rejected. Since their presence in the menu of possibilities added little information, these lump sums were dropped in later editions of the survey.

### *A. Exponential and Hyperbolic Discount Parameter Estimates*

The first two pairs of columns in Table 2 detail the discount parameter point estimates and asymptotic t-test statistics<sup>22</sup> for heteroscedastic variants of the conventional exponential discounting,  $\phi_e(t)$ , and simple hyperbolic discounting,  $\phi_h(t)$ , in the sense of Harvey (1986).<sup>23</sup> The third pair of columns provides results for the generalized hyperbolic discounting model,  $\phi_g(t)$ .

The maximized values of the log-likelihood functions for all three models are negligibly different. With only one real exception, the variables that account for contextual and individual heterogeneity bear coefficients of the same signs, and are statistically significant at approximately the same levels, across all three specifications. Thus it is possible to review most of these results generically. The results we describe in this section pertain to the logarithms of the systematically varying  $r_i$  parameter in equation (11) and the logarithm of the systematically varying  $\beta_i$  parameter in equations (12) and (13). The parameter  $r_i$  is what economists generally recognize as “the” discount rate. The  $\beta_i$  does not have this same interpretation, but like  $r_i$ , serves to define the size of the discount factor in the hyperbolic discounting specifications.

### **Fitted Discount Parameters**

#### *Effects of Choice Scenarios*

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<sup>22</sup>The final column of Table 2 displays the  $R^2$  values for auxiliary regressions where each regressor in the group is employed successively as the dependent variable in a model that uses the others in that particular index as explanatory variables (a good indicator for multicollinearity). These statistics are provided only for variables that were not part of the randomized design of this study.

<sup>23</sup>Estimation was accomplished using Matlab 6.5.0.180913a, Release 13. Since respondents were asked to react to multiple different lump sums, there are 3595 choices corresponding to 2-level answers, 3749 corresponding to 3-level answers, 3499 corresponding to 4-level answers, and 4065 corresponding to 5-level answers. We will not, in this paper, pursue the panel aspects of the data set. The sets of lump sum payment alternatives presented to each individual are assigned randomly across individuals and the offered amounts are entirely exogenous. Any unobserved heterogeneity bias has been minimized by the degree of randomization that is present in the model. Given the nonlinearity of the model, however, there may be some gains from panel methods stemming from the monotonic ordering of the lump sums, despite the randomized assignment of their sizes and the direction of this ordering. We do not pursue the further complexity of panel methods in this study.

The choice scenarios presented to individuals were randomized in terms of the size of the annual payments that were being proposed and the number of future years over which these payments would be made, so there can be no correlations among them or with other variables. In our sample, the larger the annual payments being considered, the higher is the estimated discount parameter for the individual. This result contrasts sharply with the experimental results discussed by Thaler (1981), Benzion et al. (1989) and Loewenstein and Thaler (1989). The existing literature suggests that people may be more willing to wait a year for “\$150 then versus \$100 now” than they are to wait a year for “\$15 then versus \$10 now.” Shefrin and Thaler (1988) suggest that large future amounts may be viewed merely as foregone savings interest, whereas smaller amounts may be viewed as foregone consumption, which may be more highly valued. One difference between this study and other studies, however, is that our time horizons for these annual payments are either 20, 30 or 40 years—far longer than the time horizons generally considered in laboratory experimental settings.

Our results concerning the effects of the time horizon constitute the only appreciable difference between these alternative discounting specifications. Unlike most previous experimental work, our subjects do not face a choice between one payment now versus one payment at some time in the future. Instead, they must choose between one payment now and a stream of annual payments starting now and continuing for some number of years into the future. In our exponential discounting specification, the longer the time horizon over which the lottery winnings are to be paid, the smaller is the estimated exponential discounting parameter (controlling for age).

This lower exponential discount rate for longer time horizons is consistent with the anomalies observed elsewhere in the discounting literature, and is the usual argument for resorting to a hyperbolic discounting specification. In our hyperbolic discounting specification, however, a longer time horizon leads to a very statistically significantly *larger* implied hyperbolic discount parameter. Neither model, therefore, produces discount parameters which are in-

dependent of the time horizon in question. Either this is a new empirical insight, or it has not been emphasized in the existing literature.

### *Effects of Respondent Characteristics*

Warner and Pleeter (2001) find that individual discount rates decline with age and education and that this confirms similar findings by Gilman (1976) and Black (1984) in earlier military studies. In our sample, discount rates appear to be *larger* for individuals in this sample who are older (controlling for educational attainment). Older students tend to be present in the sample because they did not pursue college education at the same time as their peers. A higher discount rate may have led them to forego college when they were younger because the greater future earnings were more heavily discounted.<sup>24</sup> Age is also negatively correlated, although not perfectly, with remaining life expectancy, which is not a factor included in our tabulated results.

The rate at which the logarithm of fitted individual discount rates increases with age (or, becomes lower with greater life expectancy) is much greater for males than for females. At 17.9 years of age, females tend to have discount rates that are roughly equal to those of similar males, but beyond that age, discount parameters for females rise much more slowly. (Warner and Pleeter (2001) identify statistically significant gender effects only in their sample of enlisted personnel, where males display higher discount rates.) Our estimated age effects also conflict with the speculation of Chesson and Viscusi (2000) that the young may be inclined to discount the future more heavily, leading to “temporal myopia” with respect to longer-term prospects.<sup>25</sup> However, we do find that the greater the individual’s educational

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<sup>24</sup>Of course, it is not possible in a cross-sectional sample to determine whether this is an age effect or a cohort effect or some combination of the two.

<sup>25</sup>We did make some effort to distinguish life expectancy from age. Our survey elicits life expectancy as a separate implicit variable by questioning respondents about whether they expected to be alive at each of each of a number of decadal intervals in the future. Crude proxies to control for life expectancy did not make any significant improvement to our models, nor did they lead to any appreciable change in the apparent effects of age on discount rates. In such a predominantly young sample, mortality risks of any kind may not yet be registering.

attainment, the lower their discount rate, consistent with the results of Harrison et al. (2002). This suggests that more highly educated individuals are more willing to postpone income, which is intuitively plausible. Students with greater educational attainment will have self-selected to be in college longer, foregoing current earnings, which may imply that they have lower discount rates.

Curiously, discount rates in our study are also larger, rather than smaller, for individuals with higher family incomes, suggesting that “impatience” in this sample is greater when the subject stems from a higher-income background. This contrasts with the reported findings of Gilman (1976) and others, and the findings of Hausman (1979), where the discount rate decreases with increased income. Higher discount rates for the poor are attributed to “uncertainty of their income streams and their lack of savings.”

Similarly perplexing results emerge for our capital access variable. The more money the individual believes they could borrow without collateral, the higher their discount rate. This is inconsistent with the results of Harrison et al. (2002) who find that discount rates are higher for individuals who believe that the chances of them getting a credit card or line of credit approved by a bank are poor (i.e., less than 75%). The “capital access” question in our survey was worded as follows: “The largest amount of money that I believe I could currently qualify to borrow from a bank, credit union, trust company or family member (without collateral) is:” The options, and percents of the sample selecting each option, were \$0 (5 percent), \$100 (4 percent), \$100 (31 percent), \$10,000 (44 percent), \$50,000 (10 percent), \$100,000 (2 percent), more than \$100,000 (4 percent).<sup>26</sup> From a population of mostly college students, where 40 percent of the sample reports a level of family income lower than \$50,000, there is some question as to whether every respondent fully understood the idea of “without collateral.” It is conceivable that family sources for particularly affluent

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<sup>26</sup>The open-ended category was arbitrarily coded as \$150,000 to produce a continuous variable to proxy for the individual’s subjective access to capital markets.

students could provide capital access at the level indicated by some of them. Although the simple correlation between the family income brackets and the capital access variable was only 0.18, more than half of the respondents who indicated the highest category for capital access also indicate the highest category for family income.

In the revealed preference context of Hausman (1979), individual discount rates are derived from consumer's choices about spending money, whereas here, they are derived from individuals' choices about how to receive money.<sup>27</sup> In the present case, capital market constraints that might be binding on purchase decisions may have much less of an effect on choices about the time pattern of receipt of money. If, compared to other people, a respondent was aware that they could borrow money at a lower effective interest rate, they should be less inclined to take the immediate lump sum and more inclined to favor the program of deferred payments, implying a smaller discount rate. Such logic does not seem to apply here. A larger discount rate might be associated with a greater perceived "need" for money in the present, or greater "impatience" about receiving money. One is left to speculate upon the distribution of desires for immediate gratification, or variations in the sense of entitlement, across college students from different socioeconomic backgrounds.

The nature of the individual's education also has some systematic effects on discount rates. In Warner and Pleeter's (2001) military sample, personnel in the Engineering and Scientist or Professional categories display lower discount rates than others, as did enlisted personnel in the top two "mental groups" with the highest test scores. Higher test scores are argued to reflect better capacity to understand the implications of intertemporal choices. In our study, we distinguish those individuals who major in social science disciplines (which will include economics) and those who major in business. A separate dummy variable (econ course) distinguishes between non-economics social science majors and those with some ex-

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<sup>27</sup>Thaler (1981), however, finds that discount rates imputed from choices about when to receive money appear to be larger, on average, than discount rates derived from choices about when to pay money.



posure to economic thinking, as well as capturing economics courses taken as an elective by students from other disciplines. Business majors and those with economics training have statistically significantly lower discount rates, and non-economist social science majors have higher discount rates.

It is somewhat surprising that political ideology along a simple liberal-conservative spectrum has far less of an effect on implied discount parameters, but subjects who identify as being either somewhat or very conservative or liberal have smaller discount parameters than subjects who identify themselves only as "moderate". If the subject is part-time or full-time employed ( $work_i = 1$ ), the estimated discount parameter is significantly larger. Perhaps students who are not patient enough to defer some types of consumption until they have completed their education are also more likely to work during their schooling. These results are similar to those of Harrison et al. (2002) who find that discount rates are statistically significantly smaller for unemployed persons.

#### *Effects of Elicitation Formats*

All of the experimental "treatments" described in this section were randomly assigned across respondents. If the answer options are arranged horizontally (left to right) from "Yes" to "No," the fitted discount parameter is statistically significantly larger. If there is an "upper left" tendency in people's choices presenting "Yes" as the first answer option may predispose the subject to be more likely to choose it, meaning they are more likely to say they prefer the lump sum, regardless of its amount, and their discount parameter will therefore appear to be larger. This is a useful result to consider in the design of such instruments in the future.

It does not seem to matter, however, whether the lump sums are presented in increasing or decreasing order. If it did, this would suggest "starting point effects." If anything, when the first lump sums in the individual's list are smaller than later lump sums, the implied discount rate may be smaller (but this effect is not remotely significant).

There is little action in terms of the number of different lump sums that the respondent is asked to consider if that number is seven or less. However, subjects who received the most complex task, with thirteen lump sums, made choices that imply discount parameters that are statistically significantly larger. (As always, one suspects a “fatigue” or “complexity” effect when the number of choice tasks increases.)

The final factor is the number of answer options provided (from 2-level choices to 5-level choices). The 3-level and 5-level choice formats each involve a “Not Sure” option, and discount rates implied in these two subsamples are statistically significantly lower, and by a comparable magnitude, compared to the 2-level and 4-level formats where the respondent was forced to make a Yes/No distinction. Note that discount rates are lower for everyone when the choice format has a “Not Sure” option, not just for those individuals who may choose this option. The 4-level format leads to estimated discount parameters that are insignificantly smaller than those produced by the 2-level format.

## **Heteroscedasticity**

### *Effects of Choice Scenarios*

Harrison et al. (2002) note in an appendix to their paper that “at lower stakes subjects are likely to expend less cognitive effort on getting the comparison right.” The greater the effort applied to the choice problem, the less noise there probably ought to be in the choices the subject makes. When we include the size of the annual payments as a factor that is eligible to shift the error variance in our model, we find their effect is hugely statistically significant and positive. Within the range of stakes in the choices in this study, then, higher stakes contribute to noisier, rather than more precise, choices.

We also allow the error dispersion on our choice models to differ systematically with the number of years over which the annual payments would be made. Choices are also noisier, the longer the time horizon involved (and this result is derived while controlling for the subject’s current age).

### *Effects of Respondent Characteristics*

In a major innovation compared to earlier empirical studies of discount rates, we employ a variety of variables to shift the logarithm of the dispersion parameter,  $\kappa_i$ , (the inverse of the “scale factor” examined in some stated preference research (see Swait and Louviere (1993) or DeShazo and Fermo (2002))).

A priori, one might expect that the ability of the individual to make a coherent decision about how they would prefer to take their lottery winnings may depend to a certain extent on their familiarity with state-sponsored lotteries. Most state lotteries offer an option like ours when large prizes are involved. Even if the individual has never won a lottery, just playing the lottery or hearing advertisements for their state lottery may invoke speculation about what they would do if they did win. For our sample, the error dispersion in the random utility model is statistically unaffected by the presence of a state-sanctioned lottery unless the individual is not eligible to play, whereupon their choices are significantly less noisy. This status, however, is a proxy for youth, so these results effectively show that exposure to a state lottery reduces error variances for the youngest respondents. This seems plausible. Error dispersion, however, seems unrelated to the number of times the respondent actually plays the lottery.

While subjects who have taken at least one course in economics have lower discount rates, they are indistinguishable from other subjects when it comes to the noise in their choices in the discounting experiment. Older subjects in this sample, however, make noisier choices, while females make choices that are less noisy (although this effect is significant only at the 10% level).

### *Effects of Elicitation Formats*

We find that choices are noisier when (a) individuals are allowed to express more uncertainty about their choices (i.e. when they are allowed to say either “probably” or “definitely” yes or no; (b) when there were more lump sums to consider; and (c) when the sizes of the

lump sums increased down the page (although only at the 10% level of significance). It is not possible to discern any significant effect upon noise of the presence of a “Not Sure” option, although the noise parameter may be slightly smaller for five-alternative answer options than for the four-alternative variant.

### *Effects of Respondent Performance*

The quality of choice information provided by a given respondent is also suspected to differ with the amount of care and attention devoted to the particular choice task.

Respondents who eventually attained the penultimate “Debriefing” page of the survey were asked about the extent to which they had to rush to complete the survey. Possible answers were 0=no, 1=yes, a little, and 2=yes, a lot. Respondents who went on to complete the survey have systematically larger error dispersions for their choices than subjects who quit after the discounting page but did not make it to the end of the survey. If the individual made it to the end of the survey, so that we have the subject’s report about how much they had to rush to finish it, the need to rush a little through the survey actually seems to lessen the noise in the choices. One might initially think that a respondent who is rushing through the survey might be more inclined to just check one column, but low variance in observed choices is not the same thing as low variance in the errors.<sup>28</sup>

We also track the entry and exit times for each page of the online survey, which allows us to construct durations for each page. In a small fraction of cases, these durations are excessively long, suggesting that the respondent was diverted to some other task while the page was still open. We examined the marginal distribution of durations for the lottery winnings page and determined that durations up to approximately the 90th percentile of this distribution appeared to be good, whereas somewhere beyond the 90th percentile, inconsistently large outlying values begin to be observed. We thus create a dummy variable to designate

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<sup>28</sup>Appendix III describes a model that explores the factors which explain decisions to check all the same column (or, in some cases, to get the choice backwards by failing to understand the headings on the choice matrix).

“good” durations on the lottery winnings page. Given that the individual’s duration on the discounting page was not excessively long (i.e. given that we measured a “good duration” rather than one likely to have been contaminated by some outside distraction that took the subject away from the task), more time spent on the choice task decreases the noise in the choices. However, excessively long times spent on this page (good duration=0) correspond to increased noise. Note that we control for the number of lump sums and the number of answer alternatives offered in estimating this effect.

### **Incidental parameters**

The ordered logit threshold parameters make up the remaining eight parameters estimated in each model. The expected signs of these thresholds should be (-, +, -, +, -, -, +, +). All point estimates have the correct signs. These thresholds are normalized on zero as the boundary between “Yes” and “No.” See the log-likelihood function developed in Appendix II for a more thorough explanation of these threshold parameters.

### **Generalization parameter for the hyperbolic model**

The maximized log-likelihood values for the heterogeneous exponential and simple hyperbolic discounting models are extremely close, differing by less than two points. If we relax the assumption of  $\gamma = 1$  in the hyperbolic discounting model, the more general model also subsumes the exponential model when  $\gamma = 0$ . The third column of results in Table 2 reveals that the more general model produces an estimate of  $\gamma = \exp(-17.12)$ . since we use a transformation that constrains the point estimate of  $\gamma$  to be strictly positive. The estimated value of  $\log(\gamma) = -17.12$  is strongly statistically significantly different from zero, with an asymptotic t-test statistic of -4.28, so  $\gamma = 1$  (and therefore the simple hyperbolic discounting model) is unequivocally rejected by our data.

#### *B. Range of Estimates and Precisions in Fitted Individual Discount Parameters*

Frederick et al. (2002) review estimates of individual discount rates in the literature that

range from zero to infinity. What do our models imply about the distributions of fitted empirical discount factors, from both the exponential and the hyperbolic models, for this sample? The discussion in the previous section concerned the logarithms of the underlying fitted discount parameters. In this section, we convert our estimates into the implied time profiles of discount factors. This is useful in that it permits one to visualize the extent of the heterogeneity in discounting behavior that we have uncovered in our sample.

We estimate  $r_i = \exp(r'Z_i)$  and  $\beta_i = \exp(\beta'Z_i)$  by maximum likelihood methods, so each of the discount parameter “indexes,”  $r'Z_i$  and  $\beta'Z_i$ , can be assumed to be asymptotically normally distributed. The process of exponentiation takes the symmetric normal distribution and converts it into a skewed lognormal distribution. To produce expected values for each individual fitted discount parameter, therefore, it is first necessary to calculate the estimated variances of these linear combinations. This task requires the relevant submatrix from the overall parameter variance-covariance matrix computed at the converged values of the full set of model parameters. We use these variances (for the relevant linear combinations of parameters and data) in calculating the expected values of each estimated discount parameter:  $E[r_i] = \exp(r'Z_i) \exp(\sigma_{r_i}^2/2)$  and  $E[\beta_i] = \exp(\beta'Z_i) \exp(\sigma_{\beta_i}^2/2)$ . For any particular vector of explanatory variables, an approximate 95 percent asymptotic confidence interval for the systematically varying index can be calculated by exponentiating the calculated values of  $r'Z_i \pm 1.96\sqrt{\sigma_{r_i}^2}$ . Transforming these confidence bounds by exponentiating will produce an approximate 95 percent confidence interval for the implied individual discount parameter for each subject in our sample.

The next step is to display the range of different discount parameters evidenced in our data. Figure 2a displays a double plot for the exponential discounting model. The lower inverted histogram describes the marginal distribution in the sample of fitted expected discount parameters,  $E[r_i]$ , for the  $i = 1, \dots, 2062$  respondents in our sample. Keep in mind that all of the variability in these point estimates across the sample is created by differences

in the choice scenarios posed, differences in respondent attributes, or differences in the mode of elicitation of the choice. The upper plot translates each point estimate measured along the horizontal scale to a vertical scale that displays both the point estimate and the fitted individual confidence bounds that go along with it. This is a non-standard mode of exposition, but it reveals the extent of non-overlapping confidence intervals for the fitted exponential discount rates for different individuals in our sample. Figure 2b displays analogous results for the hyperbolic discounting model, and Figure 2c gives the results for the generalized hyperbolic specification.

The bottom panel of Table 2 displays some descriptive statistics for the marginal distributions across the sample of the fitted point estimates for each type of discount parameter. These differences across the sample are due to the different characteristics of respondents and the different experimental choice scenarios and elicitation methods to which they were exposed. Our fitted individual exponential discount parameters (with the familiar interpretation of “discount rates”) range between about 2 percent and 20 percent, and our fitted hyperbolic discount parameters vary between about 0.17 and 0.87. In the generalized hyperbolic model, which most closely mirrors the results for the exponential model, our fitted discount parameters again range between about 2 percent and 18 percent. Considerable heterogeneity in fitted discounting behavior therefore exists, and this heterogeneity is likely to be economically significant, as well as statistically significant.

Finally, it is helpful to visualize the different implications of our fitted exponential, hyperbolic, and generalized hyperbolic models. Figure 3 compares the time profiles for the sample median values of the discount parameters for the exponential, simple hyperbolic and the generalized hyperbolic models. As expected, the exponential and generalized hyperbolic models coincide. Figure 4 displays the implied discount-factor time profiles for selected percentiles of the marginal distribution in our sample of the fitted simple hyperbolic discount parameters. Finally, Figure 4 shows the distribution of discount factor profiles for the gen-

eralized hyperbolic model (using the single point estimate of  $\lambda$ ). As expected, based on our parameter estimates, these match the results for the exponential model almost exactly, so we do not provide a separate figure for the exponential model. There appears to have been no previous effort to estimate such comprehensive empirical models that comparing both the exponential and hyperbolic model with the generalized hyperbolic specification. For these data, it seems that the more-general hyperbolic model may favor the pattern of discounting behavior implied by the exponential model.

This result is at odds with the general sentiment in the recent literature that the hyperbolic discounting model is widely better at explaining choices than the exponential model. This finding may stem from the choice contexts we employ in our study. Other than the Warner and Pleeter (2001) military study, most comprehensive empirical work that attempts to distinguish between exponential and hyperbolic discounting seems to have employed choices between a single payment now and a single payment at some point in the future. In experimental contexts, this future payment cannot be too remote. In our choice contexts, individuals are asked to choose between a single payment now and a stream of payments each year, for either 20, 30 or 40 years into the future. Thus our choice context includes payments in the near-term as well as the long term, but the durations of the hypothetical delayed payment program are atypically long compared to those used in other studies.

## V. Conclusions

We opened this paper by noting the continuing debate over the correct single social discount rate to be used in assessing the efficiency of policy choices involving public goods. This debate stems, in part, from insufficient knowledge about the nature of heterogeneity in individual time preferences. When policy choices confer different net benefits with different time profiles on different constituencies that may have different time preferences, heterogeneity



in individual discount rates is an important consideration. Our results add to the mounting evidence that different types of individuals have different discount rates, and individuals also discount in systematically different ways according to the choice contexts they face (amounts and time horizons) and the manner in which those choices are posed.

To evaluate a particular policy that affects a particular population, it might be possible to do better than simply to employ a single one-size-fits-all social discount rate. Suppose that for this public good, discount parameters could be selected for a choice context that approximately matches the the sizes and time profiles of the net benefits faced by each individual. It might then be possible first to discount the time profile of net benefits for each individual in the affected population, and then to aggregate these present-value net benefits across individuals. Of course, there will still remain other important considerations, including the necessity to recognize the interests of future generations and others who are not part of the current franchise. But if we have a model that captures individual heterogeneity in discount rates, we can use this model to simulate counterfactual distributions of discount rates (perhaps a distribution that preserves this heterogeneity, but shifts the distribution downwards).

We motivated our utility-theoretic effort to quantify the heterogeneity in individual discount rates by noting that any individual's tastes must encompass time preferences just as they do preferences across contemporaneous bundles of goods. Previous efforts to demonstrate individual heterogeneity in discount rates have used reduced-form specifications in their empirical analysis. The random utility choice models we employ in this paper lend themselves naturally to a seamless integration of discounting choices with other choices made by the same individuals. When discounting choice data are pooled with data for some other choice (for example, concerning a consumer durable or a public good), it may be possible to assume that these choices are fundamentally linked. If the "other" choice involves different time profiles of benefits and costs, it must therefore also involve discounting. Individual

discount rates, if not identical for both types of choice, may be proportional. The extra information about the time dimension of preferences (contained in the explicit discounting choices) can be used to better identify the other distinct preference parameters involved in the “other” choice.<sup>29</sup>

When building a model that includes not just the discounting component described in this paper, but also another choice, a number of econometric considerations will be relevant. If alternatives within the other choice involves differences in net income, the marginal utility of income can be constrained to be identical across the discounting choice and the other choice. We should also expect that the error variances may differ substantially across these two different types of choices. Depending upon the cognitive burden of each choice context, one type of choice may be noisier than another. The factors that appear to contribute to error variances in the discounting choice portion of the model may affect error variances for the other choice in a different manner. It can be difficult to estimate discount rates simultaneously with other dimensions of preferences without a separate source of information about discounting behavior, which is why we advocate an auxiliary specific discounting question like that analyzed here. However, this also means it may be very challenging to test the maintained hypothesis that people use the same discounting parameters for both types of choices.

With respect to the discounting models explored in this paper, it has been well-documented that empirically estimated discount rates vary dramatically across samples and across the choice contexts and techniques used to elicit them (Frederick et al., 2002).<sup>30</sup> Now, within this one study, we have varied a wide array of the factors that have elsewhere differed across

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<sup>29</sup>One of the authors is currently experimenting with this formal simultaneous estimation strategy in another project, using a different data set. While this other data set is a representative sample of the U.S. population, there is no variability in elicitation formats, so it is not possible to assess the full range of design effects covered in this paper.

<sup>30</sup>With at least forty empirical measures of discount rates, it is somewhat surprising that no researcher has yet attempted a regression meta-analysis that can shed light on how the characteristics of a particular study contribute to higher or lower estimated values of the discount rate.

studies, allowing us to make systematic assessments of the effects on implied individual discount rates of the choice context, the format of the elicitation method, and individual characteristics. At the margin, our fitted individual discount rates range between 2 and 20 percent (for this sample, and for the range of choice contexts considered in this study), with a sample mean of roughly 6.4 percent. The Warner and Pleeter (2001) military study produces estimates of discount rates that range between 0 to over 30 percent. The field experiment of Harrison et al. (2002) produces an overall individual discount rate of 28.1 percent for Denmark, whereas an analogous laboratory experiment by Collier and Williams (1999), using U.S. college student, produces estimates on the order of 15 to 18 percent.

Like Harrison et al. (2002), our study offers exogenous randomization in the choice contexts (amounts and time horizons). In Warner and Pleeter (2001), decision-makers face a different lump sums, annual annuity payments, and time horizons for these payments, but these conditions are endogenous since they are determined by the individual's own base pay and years of service, which may be correlated with their individual discount rates. We are able to explore systematic differences in implied discount rates as a function of exogenous differences in the amounts of money at stake and the time horizons over which the payments are being considered.

These exogenous variations in the elicitation format, however, come at the cost of our reliance on stated, rather than revealed, choices. Harrison et al. (2002) make the choices more "real" for their subjects by using a field experiment, where just one participant in each group is selected to receive a real payoff. However, the desire to include these real payoffs, constrained by a research budget, limits their sample size and the time horizons that can be assessed. Our stated choice framework allows our study subjects to consider the much longer time horizons likely to be relevant for many types of policies. The fact that each participant faces some uncertainty about whether they will receive a real payoff also means that uncertainty may confound the elicitation of discount rates (as in Chesson and Viscusi,

2000).

In terms of different elicitation formats (for common choice scenarios regarding amounts and time horizons), there are no experimental treatments at all in Warner and Pleeter (2001) or Harrison et al. (2002). We are able to examine the systematic effects of differences in the number of choices made by each individual and the number of answer options offered for each choice, whether answer options are listed from “yes” to “no” or vice-versa, and whether the offered lump sum payments are arranged from largest to smallest or vice-versa.

The military records used by Warner and Pleeter (2001) provide a wealth of detail about each individual, including gender, ethnicity, wage rate, age and education level, dependents, test scores and area of specialization. However, this detail comes at the cost of relying upon an all-volunteer military-only sample. For the survey used in the present study (as for Harrison et al. (2002)), the individual characteristics are self-reported. This places a soft constraint on the amount of detail that can be elicited because of limits on survey length. Although we have fewer individual characteristics in our data set than Harrison et al. (2002), their limited sample size results in only four statistically significant coefficients on the twenty individual characteristics variables they examine. Thirteen of the fourteen variables we employ as systematic shifters of our discount parameter estimates are statistically significant and we offer previously unavailable results including those for gender and age interactions, the effects of prior training in economics, and the effects of political ideology.

With one choice per individual, Warner and Pleeter (2001) are able to use a censored normal regression model to estimate the slope parameters of their atheoretic discount rate formula. Harrison et al. (2002) also use an atheoretic discounting specification. They rely upon packaged algorithms for interval data, using the transition between “yes” and “no” answers to a progression of choices involving different implied discount rates to identify the interval wherein the implied discount rate lies. The complexity due to our use of different elicitation methods necessitates that we use some fairly complicated maximum likelihood

algorithms to estimate our parameters when we pool the data across all individuals and wish to test for differences in discount parameters across elicitation methods. In anticipation of integrating this analysis with other types of choices in a random utility specification, we also preserve a utility-theoretic framework, basing our analysis on the typical assumption of a logistic distribution for the underlying utility-difference.

Unlike any earlier analyses, our models also allow for heteroscedasticity (different error variances in the utility-differences that drive individuals' choices between the payment options). There is some informal speculation in the literature about what features of the choice context might cause subjects to expend less cognitive effort on comparisons among alternatives. We treat the observed heteroscedasticity as symptomatic of differences in choice consistency and find a number of statistically significant determinants of this heteroscedasticity, including some features of choice context and elicitation format, as well as individual characteristics.

As for Warner and Pleeter's (2001) military sample, the distribution of discount rates in our multi-university and international student sample is not likely to be representative of the distribution of discount rates in the general population because of self-selection of individuals into these groups. Nevertheless, insights about how discount rates vary with all of our factors, within our broad college population, are new. This information may be specifically useful, for example, in helping policymakers concerned with tertiary education to understand the different choices made by different types of students.

The direct elicitation of individual-specific discount rates will always be problematic because ordinary citizens cannot be expected to know what a discount rate is, nor can they be expected to have introspected very carefully about the magnitude of their own individual rate. As has been the case since some of the first empirical efforts of Hausman (1979), researchers will typically be forced to infer implicit discount rates from either real or stated choices. Only a sample like that of Weitzman (2001)—professional economists—could

reasonably be expected to articulate a discount rate directly with any degree of understanding of the concept.<sup>31</sup>

The empirical results presented in this paper also constitute a specific illustration of one type of elicitation strategy (how to take some hypothetical lottery winnings) that holds promise for helping us understand the determinants of heterogeneity in individual discount rates in a wide variety of other choice contexts. Considerably more general-population empirical work will be needed before we are able to quantify individual-specific discount rates with sufficient accuracy to warrant the calculation of the present discounted value of a stream of future net benefits by first discounting individually or for distinct groups, then aggregating.<sup>32</sup> We do, however, advocate a shift in the direction of “discount first, aggregate second” as a research goal.

With respect to Weitzman’s (2001) findings for his survey of economists, it is interesting to note that we find private discount rates differing with age and/or life expectancy, gender, income, access to capital, and with exposure to economics training, among other things. Perhaps Weitzman’s “leading economists” subsample displays slightly higher discount rates than the broader population of economists simply because the individuals on Weitzman’s list are older and exclusively male. They are also likely to have higher family incomes and greater access to capital. But we must also consider whether Weitzman’s sample of economists should have the prerogative of dictating the social discount rate to be used in policy-making, especially when it seems that those who have taken a course in economics exhibit statistically significantly lower discount rates. As in other studies of how economists are different, though, there remains the question of whether economic training leads to lower discount rates, or whether individuals with innately lower discount rates self-select to become

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<sup>31</sup>Weitzman was persistent in pressing his respondents to provide a point estimate of the relevant social discount rate for climate change policy.

<sup>32</sup>We have argued that this is the conceptually correct approach, rather than first aggregating net benefits in each future period, then discounting using a single representative discount factor.

economists.

What about the choice between exponential and hyperbolic discounting? Much has been made, over recent years, of the apparent superiority of the hyperbolic discounting model for explaining declining impatience with respect to longer-term tradeoffs. For this particular data set, however, the exponential model is very narrowly better than the simple hyperbolic model in terms of its ability to explain individuals' stated choices. However, the difference in the log-likelihood function is less than 2 (for 14,908 total choices). When we generalize the simple hyperbolic specification (with heterogeneity and heteroscedasticity) by freeing up a single additional shape parameter, this shape parameter is very close to zero (where, in the limit, a zero value corresponds to exponential discounting). The fitted discount factors revert to match the exponential model almost exactly. This outcome suggests that the standard exponential model and the alternative generalized hyperbolic discounting model explain these observed choices about equally well. This result contrasts with the growing amount of evidence that argues in favor of hyperbolic discounting. In this particular case, the familiar exponential model seems to be a sufficiently adequate representation of preferences.

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Table 1 - Descriptive Statistics (n = 2062)

| Abbreviation  | Description  | Mean    | Std.Dev. |
|---|--|---------|----------|
| <i>Nature of the choice scenario (randomized)</i>             |  |         |          |
| annual amount   | annual lottery payment (\$ 000)                        | 2.144   | 1.629    |
| time horizon  | number of annual payments                              | 29.91   | 8.341    |
| <i>Individual-specific characteristics (non-orthogonal)</i>   |  |         |          |
| age   | age (years; midpoints of 5-yr brackets)                | 22.24   | 5.502    |
| female  | =1 if female, =0 if male                               | 0.5024  |          |
| Have college years?   | =1 if have data on college years completed             | 0.9675  |          |
| College years   | # years completed if available and $\leq 4$            | 1.740   | 1.442    |
| College years >4  | =1 if college years available and >4                   | 0.1499  |          |
| family income   | family income (\$ 000, bracket midpoints)              | 67.00   | 38.89    |
| capital access  | capital access (\$ 000)                                | 17.60   | 32.89    |
| major = business  | =1 if major(ed) in business                            | 0.3497  |          |
| major = soc.science   | =1 if major(ed) in social sciences                     | 0.2978  |          |
| econ course   | =1 if economics course ever taken                      | 0.8792  |          |
| liberal   | =1 if "moderately" or "very" liberal                   | 0.4316  |          |
| conservative  | =1 if "moderately" or "very" conservative              | 0.2517  |          |
| work  | =1 if work full- or part-time                          | 0.4360  |          |
| lottery (can play)  | =1 if lottery available and can play                   | 0.8337  |          |
| lottery (can't play)  | =1 if lottery available but can't play                 | 0.05723 |          |
| times played/yr   | times lottery played per year                          | 3.442   | 7.626    |
| <i>Respondent performance (non-orthogonal)</i>                |  |         |          |
| finished survey?  | respondent finished entire survey                      | 0.9326  |          |
| rushlit   | =1 if somewhat rushed to complete survey               | 0.3055  |          |
| rushlot   | =1 if very rushed to complete survey                   | 0.01649 |          |
| good duration   | =1 if duration on task in "0-90 <sup>th</sup> %ile"    | 0.9011  |          |
| duration on task  | duration (minutes) if within "0-90 <sup>th</sup> %ile" | 1.048   | 0.6768   |
| <i>Characteristics of the elicitation format (randomized)</i> |  |         |          |

|                 |                                      |        |       |
|-----------------|--------------------------------------|--------|-------|
| "yes" on left   | = 1 if "yes" on left                 | 0.5145 |       |
| increasing sums | = 1 if lump sums increasing          | 0.4864 |       |
| 5 lump sums     | = 1 if five lump sums considered     | 0.2371 |       |
| 7 lump sums     | = 1 if seven lump sums considered    | 0.2570 |       |
| 13 lump sums    | = 1 if thirteen lump sums considered | 0.2493 |       |
| # of lump sums  | continuous number of lump sums       | 6.995  | 3.746 |
| 3-level answers | = 1 if 3-level response options      | 0.2473 |       |
| 4-level answers | = 1 if 4-level response options      | 0.2342 |       |
| 5-level answers | = 1 if 5-level response options      | 0.2716 |       |

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Table 2

Parameter Estimates: Heteroscedastic Exponential, Hyperbolic, and Generalized Hyperbolic Discounting Models (n=2062)

| Variables  | Exponential         |            | Hyperbolic                  |           | Generalized Hyperbolic              |            | Aux. R <sup>2</sup> |
|--|---------------------|------------|-----------------------------|-----------|-------------------------------------|------------|---------------------|
|  | Estimate            | t-ratio    | Estimate                    | t-ratio   | Estimate                            | t-ratio    |                     |
| Marginal utility of income: $\exp(\mu)$              |                     |            |                             |           |                                     |            |                     |
| constant   | 3.348               | (12.37)**  | 3.345                       | (12.79)** | 3.347                               | (12.68)**  |                     |
| Discount parameters:                                 | $r_i = \exp(r'Z_i)$ |            | $\beta_i = \exp(\beta'Z_i)$ |           | $\beta_i = \exp(\beta'Z_i), \gamma$ |            |                     |
| constant   | -4.411              | (-11.00)** | -2.999                      | (-9.10)** | -4.397                              | (-11.33)** | -                   |
| <i>- choice scenarios (randomized)</i>               |                     |            |                             |           |                                     |            |                     |
| annual payment                                       | 0.02623             | (2.03)**   | 0.02235                     | (2.07)**  | 0.02503                             | (2.00)**   | -                   |
| time horizon   | -0.005563           | (-3.04)**  | 0.01228                     | (7.82)**  | -0.005448                           | (-3.07)**  | -                   |
| <i>- respondent characteristics (non-orthogonal)</i> |                     |            |                             |           |                                     |            |                     |
| log(age)   | 0.6120              | (4.93)**   | 0.4921                      | (4.83)**  | 0.5957                              | (4.96)**   | 0.645               |
| female   | 1.536               | (3.53)**   | 1.234                       | (3.48)**  | 1.496                               | (3.58)**   | 0.996               |
| female*log(age)                                      | -0.5307             | (-3.74)**  | -0.4261                     | (-3.69)** | -0.5167                             | (-3.79)**  | 0.996               |
| have college years?                                  | -0.1484             | (-2.06)**  | -0.1215                     | (-2.00)** | -0.1441                             | (-2.05)**  | 0.118               |
| college years  | 0.07094             | (5.06)**   | 0.05740                     | (4.90)**  | 0.06857                             | (5.04)**   | 0.518               |
| college years >4                                     | 0.1212              | (2.06)**   | 0.09594                     | (1.95)*   | 0.1170                              | (2.05)**   | 0.574               |
| family income  | 0.001411            | (3.79)**   | 0.001191                    | (3.83)**  | 0.001367                            | (3.79)**   | 0.101               |
| capital access                                       | 0.002488            | (5.24)**   | 0.002144                    | (5.61)**  | 0.002423                            | (5.30)**   | 0.132               |
| major = business                                     | 0.003511            | (0.12)     | 0.005203                    | (0.20)    | 0.003606                            | (0.12)     | 0.129               |
| major = social science.                              | 0.1644              | (5.37)**   | 0.1383                      | (5.46)**  | 0.1599                              | (5.39)**   | 0.103               |
| econ course  | -0.1797             | (-4.17)**  | -0.1581                     | (-4.42)** | -0.1747                             | (-4.19)**  | 0.129               |
| liberal  | -0.09731            | (-3.09)**  | -0.08218                    | (-3.13)** | -0.09492                            | (-3.11)**  | 0.276               |
| conservative   | -0.07064            | (-1.94)*   | -0.06864                    | (-2.26)** | -0.06929                            | (-1.96)**  | 0.266               |
| work   | 0.07844             | (2.85)**   | 0.06152                     | (2.68)**  | 0.07596                             | (2.85)**   | 0.031               |
| <i>- elicitation formats (orthogonal)</i>            |                     |            |                             |           |                                     |            |                     |
| “yes” on left  | 0.05651             | (2.07)**   | 0.05029                     | (2.21)**  | 0.05482                             | (2.08)**   | -                   |

|                 |           |           |           |           |           |           |   |
|-----------------|-----------|-----------|-----------|-----------|-----------|-----------|---|
| increasing sums | -0.006687 | (-0.24)   | -0.003128 | (-0.14)   | -0.006532 | (-0.25)   | - |
| 5 lump sums     | -0.02981  | (-0.61)   | -0.01493  | (-0.37)   | -0.027772 | (-0.59)   | - |
| 7 lump sums     | 0.05516   | (1.22)    | 0.05170   | (1.38)    | 0.05418   | (1.24)    | - |
| 13 lump sums    | 0.08292   | (1.94)*   | 0.08145   | (2.30)**  | 0.08210   | (1.98)**  | - |
| 3-level answers | -0.3176   | (-6.59)** | -0.2576   | (-6.37)** | -0.3068   | (-6.50)** | - |
| 4-level answers | -0.05769  | (-1.34)   | -0.05603  | (-1.60)   | -0.05525  | (-1.33)   | - |
| 5-level answers | -0.2484   | (-4.96)** | -0.2136   | (-5.24)** | -0.2409   | (-5.05)** | - |

Heteroscedasticity:  $\kappa_i = \exp(\kappa'W_i)$ , where intercept term in  $\kappa'W_i$  is normalized to zero

- *choice scenarios (randomized)*

|               |          |          |          |          |          |          |  |
|---------------|----------|----------|----------|----------|----------|----------|--|
| annual amount | 0.09897  | (9.44)** | 0.09829  | (9.39)** | 0.09905  | (9.45)** |  |
| horizon       | 0.005293 | (3.11)** | 0.005007 | (2.95)** | 0.005296 | (3.12)*  |  |

- *respondent characteristics (non-orthogonal)*

|                       |           |           |           |           |           |           |       |
|-----------------------|-----------|-----------|-----------|-----------|-----------|-----------|-------|
| log(age)              | 0.3295    | (4.44)**  | 0.3306    | (4.58)**  | 0.3296    | (4.53)**  | 0.045 |
| female                | -0.04606  | (-1.71)*  | -0.04636  | (-1.72)*  | -0.04628  | (-1.72)*  | 0.019 |
| any economics?        | 0.04755   | (1.21)    | 0.04504   | (1.15)    | 0.04760   | (1.21)    | 0.020 |
| lottery (can play)    | 0.03206   | (0.72)    | 0.03461   | (0.78)    | 0.03231   | (0.73)    | 0.328 |
| lottery (can't play)  | -0.2142   | (-3.07)** | -0.2136   | (-3.07)** | -0.2140   | (-3.08)** | 0.329 |
| times play lottery/yr | -0.001339 | (-0.73)   | -0.001311 | (-0.71)   | -0.001352 | (-0.73)   | 0.021 |

- *elicitation formats (orthogonal)*

|                 |          |           |          |           |          |           |   |
|-----------------|----------|-----------|----------|-----------|----------|-----------|---|
| “yes” on left   | -0.07449 | (-2.74)** | -0.07533 | (-2.77)** | -0.07458 | (-2.74)** |   |
| increasing sums | 0.04563  | (1.68)*   | 0.04117  | (1.51)    | 0.04544  | (1.67)*   | - |
| # of lump sums  | 0.02182  | (6.16)**  | 0.02193  | (6.19)**  | 0.02182  | (6.16)**  | - |
| 3-level answers | 0.04481  | (0.64)    | 0.05514  | (0.79)    | 0.04426  | (0.63)    | - |
| 4-level answers | 0.2136   | (2.98)**  | 0.2144   | (2.99)**  | 0.2129   | (2.97)**  | - |
| 5-level answers | 0.1840   | (2.67)**  | 0.1893   | (2.75)**  | 0.1840   | (2.68)**  | - |

- *respondent performance (non-orthogonal)*

|                  |          |            |           |            |           |            |       |
|------------------|----------|------------|-----------|------------|-----------|------------|-------|
| finished survey  | 0.1527   | (2.97)**   | 0.1590    | (3.10)**   | 0.1525    | (2.97)**   | 0.043 |
| rushed a little  | -0.05612 | (-1.86)*   | -0.056380 | (-1.87)*   | -0.056043 | (-1.86)*   | 0.061 |
| rushed a lot     | -0.07178 | (-0.69)    | -0.06698  | (-0.64)    | -0.07178  | (-0.69)    | 0.017 |
| good duration    | 0.3783   | (6.94)**   | 0.3735    | (6.84)**   | 0.3782    | (6.92)**   | 0.280 |
| duration on task | -0.2313  | (-10.26)** | -0.2288   | (-10.14)** | -0.2314   | (-10.25)** | 0.287 |

Ordered logit thresholds

- 3-level answers

|               |        |           |        |           |        |           |   |
|---------------|--------|-----------|--------|-----------|--------|-----------|---|
| $\alpha_{30}$ | -3.880 | (-3.58)** | -3.804 | (-3.67)** | -3.868 | (-3.63)** | - |
| $\alpha_{31}$ | 0.2040 | (0.63)    | 0.2957 | (0.89)    | 0.2135 | (0.64)    | - |

- 4-level answers (middle threshold normalized to zero)

|               |        |           |        |           |        |           |   |
|---------------|--------|-----------|--------|-----------|--------|-----------|---|
| $\alpha_{40}$ | -8.319 | (-3.69)** | -8.313 | (-3.81)** | -8.312 | (-3.77)** | - |
| $\alpha_{42}$ | 7.529  | (3.67)**  | 7.494  | (3.79)**  | 7.526  | (3.76)**  | - |

- 5-level answers

|               |        |           |        |           |        |           |   |
|---------------|--------|-----------|--------|-----------|--------|-----------|---|
| $\alpha_{50}$ | -10.11 | (-3.71)** | -10.14 | (-3.84)** | -10.11 | (-3.80)** | - |
| $\alpha_{51}$ | -3.931 | (-3.61)** | -3.952 | (-3.73)** | -3.930 | (-3.69)** | - |
| $\alpha_{52}$ | 0.1689 | (0.47)    | 0.1563 | (0.46)    | 0.1703 | (0.50)    | - |
| $\alpha_{53}$ | 6.063  | (3.54)**  | 6.061  | (3.67)**  | 6.065  | (3.65)**  | - |

Generalized hyperbolic parameter:

|          |   |   |   |   |        |           |   |
|----------|---|---|---|---|--------|-----------|---|
| $\gamma$ | - | - | - | - | -17.12 | (-4.28)** | - |
|----------|---|---|---|---|--------|-----------|---|

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|           |            |            |            |
|-----------|------------|------------|------------|
| Max Log L | -14875.817 | -14877.403 | -14875.420 |
|-----------|------------|------------|------------|

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Sample distributions of fitted discount parameters (= discount rate only for exponential model):  $E[r_i]$ ;  $E[\beta_i]$

---

|                               |   |  |   |
|-------------------------------|---|--|---|
| <i>Sample Average over i:</i> | $\exp\left(\hat{r}'Z_i + \frac{\sigma_r^2}{2}\right): 0.0642$ | $\exp\left(\hat{\beta}'Z_i + \frac{\sigma_\beta^2}{2}\right): 0.313$ | $\exp\left(\hat{\beta}'Z_i + \frac{\sigma_\beta^2}{2}\right): 0.0621$ |
| Minimum                       | 0.0284  | 0.167  | 0.0282  |
| 5 <sup>th</sup> percentile    | 0.0408  | 0.207  | 0.0400  |
| 25 <sup>th</sup> percentile   | 0.0512  | 0.258  | 0.0500  |
| Median                        | 0.0612  | 0.300  | 0.0594  |
| 75 <sup>th</sup> percentile   | 0.0727  | 0.354  | 0.0702  |
| 95 <sup>th</sup> percentile   | 0.1000  | 0.459  | 0.0958  |
| maximum                       | 0.1917  | 0.871  | 0.1804  |

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Figure 1 – One Variant of the Discounting Choice Page

**Trade-offs involving money over time**

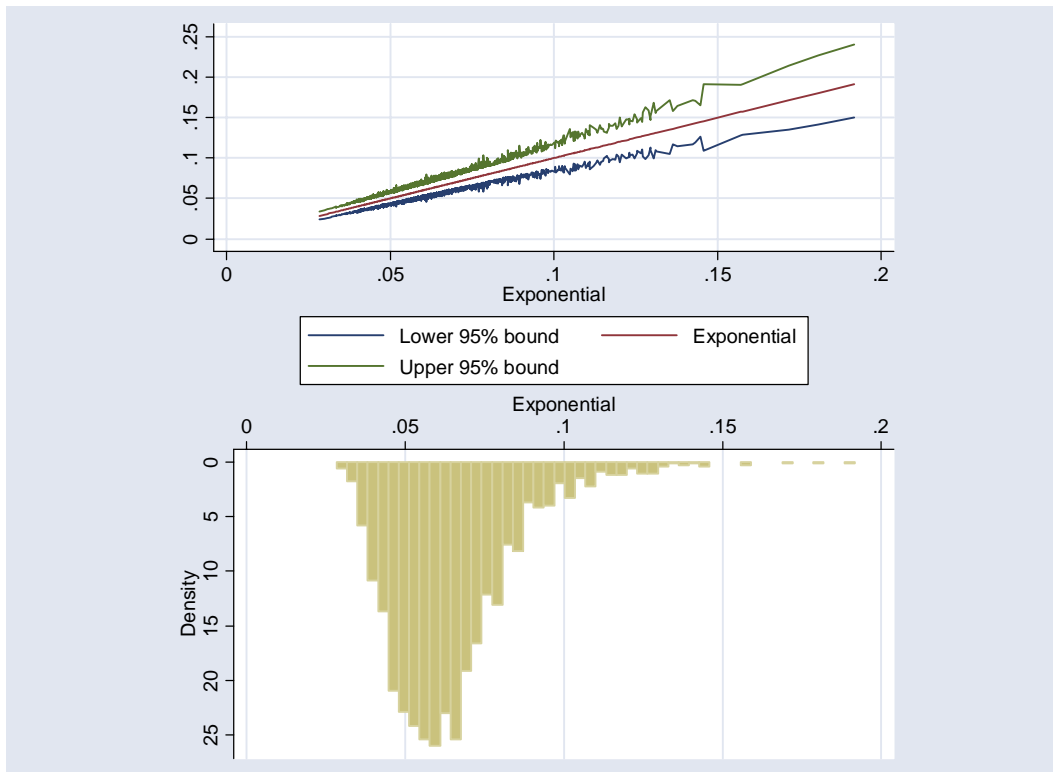
Imagine that you have won a lottery.

The lottery commission gives you two ways of taking your winnings:

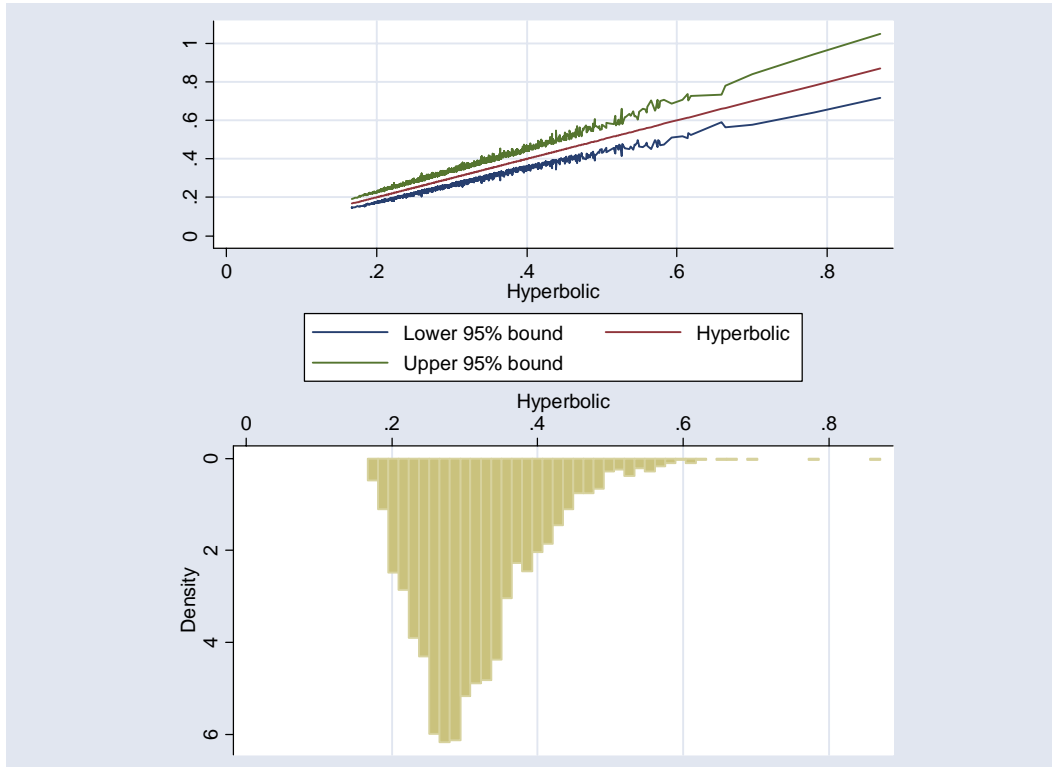
1. **\$300 each year for 40 years** (for a total of **\$12,000**), with the first payment today, OR
2. A smaller **lump sum** payment today (which you could put into a savings account or invest, or just use it to pay for something you really want or need right now).

*For each row in the table below, please click one answer button.*

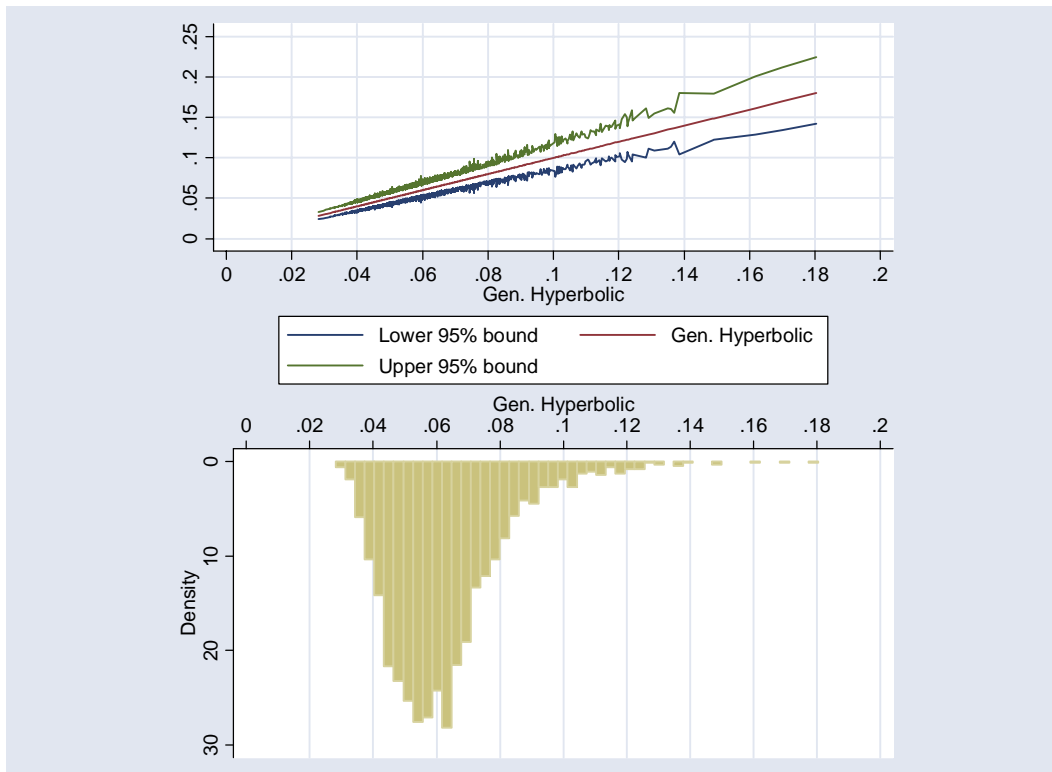
| If your lump sum payment would be: | Would you prefer this lump sum payment, rather than the annual installments? |                          |                          |                          |
|------------------------------------|--|--------------------------|--------------------------|--------------------------|
|                                    | definitely<br>no   | probably<br>no           | probably<br>yes          | definitely<br>yes        |
| \$9,900                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$7,100                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$5,400                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$4,300                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$3,500                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$2,800                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| \$1,800                            | <input type="checkbox"/>   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
|                                    | definitely<br>no   | probably<br>no           | probably<br>yes          | definitely<br>yes        |



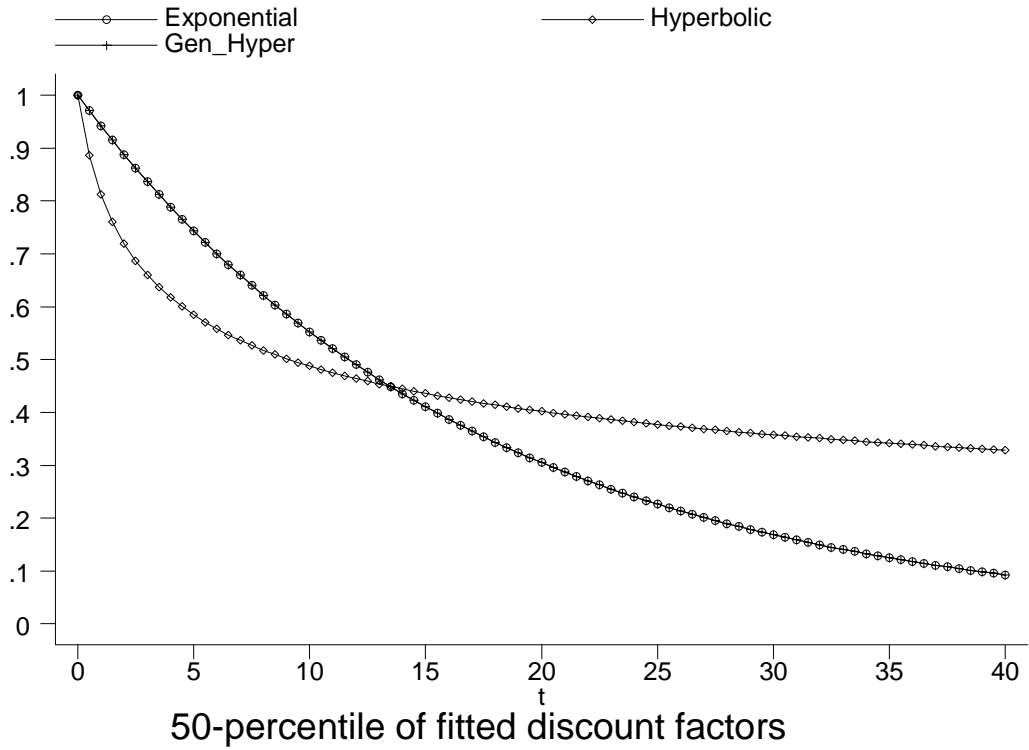
**Figure 2a** - Sample marginal distribution of fitted exponential discount parameters (lower histogram) and approximate 95% confidence bounds on estimates for point estimate (upper diagram). Confidence bounds reflect asymptotic joint normality of estimated slope parameters in the systematically varying discount parameter formula.



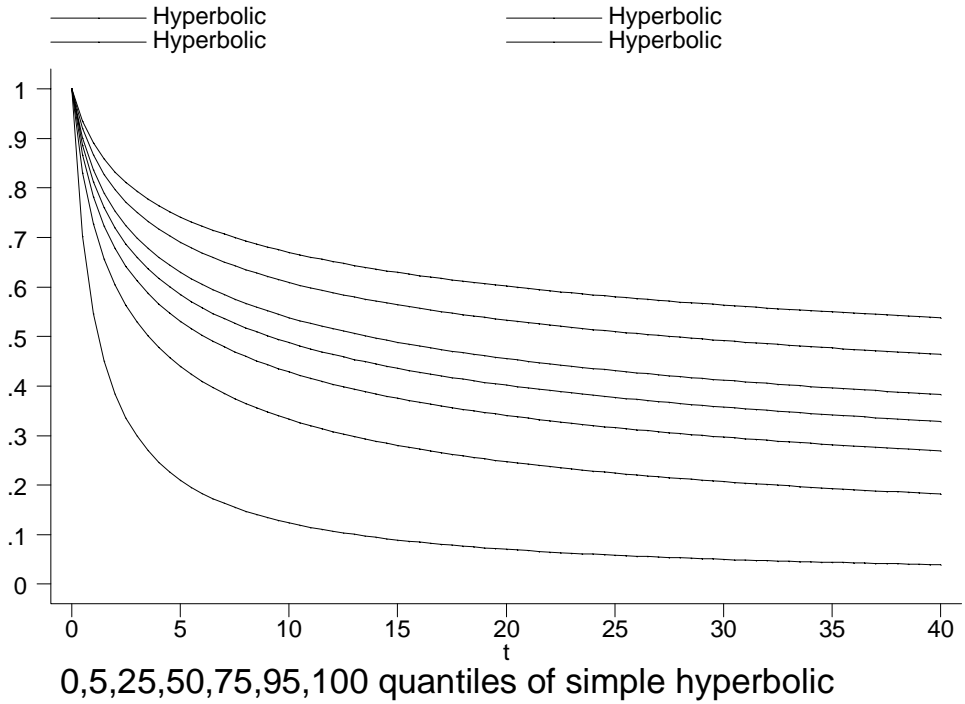
**Figure 2b:** Sample marginal distribution of fitted simple hyperbolic discount parameters (lower histogram) and approximate 95% confidence bounds on estimates for point estimate (upper diagram). Confidence bounds reflect asymptotic joint normality of estimated slope parameters in the systematically varying discount parameter formula.



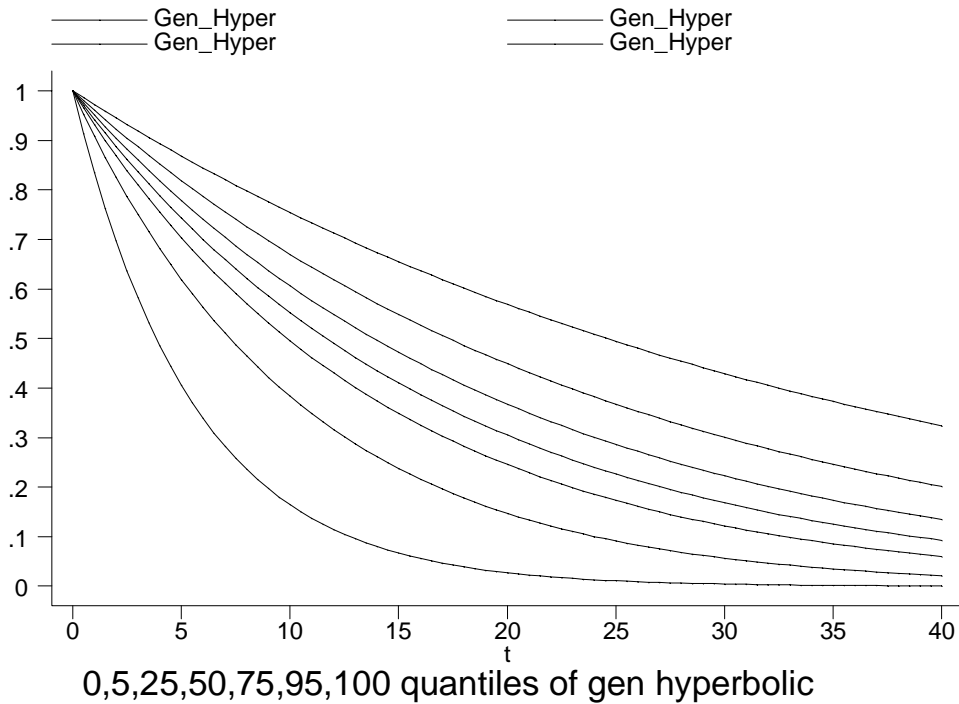
**Figure 2c** - Sample marginal distribution of fitted generalized hyperbolic discount parameters (lower histogram) and approximate 95% confidence bounds on estimates for point estimate (upper diagram). Confidence bounds reflect asymptotic joint normality of estimated slope parameters in the systematically varying discount parameter formula



**Figure 3** – Implied time profile of discount factors (years) for median fitted values of discounting parameters for our three specifications. NOTE: Exponential and generalized hyperbolic (Gen\_Hyper) models produce discount factor profiles that coincide; simple hyperbolic discount factor profile drops more quickly at first, then flattens out.



**Figure 4** –. Heterogeneity across the sample in fitted simple hyperbolic discounting models: Profiles drawn for selected quantiles of the fitted marginal distribution of expected values of the discounting parameter: Variability is due to different respondent characteristics, choice scenarios, and elicitation formats.



**Figure 5** – Heterogeneity in fitted generalized hyperbolic discounting models: Selected quantiles of the fitted marginal distribution across the sample: Variability in these profiles is due to different respondent characteristics, choice scenarios, and elicitation formats. NOTE: discount factors implied by the generalized hyperbolic model are essentially identical to the exponential model, so we do not provide a separate figure for them.

## APPENDIX I

### Gamma vs. Generalized Hyperbolic Discounting

Using the same notation as in Loewenstein and Prelec (1992), but adapting it to Weitzman's (2001) model, the formula for the individual (continuous-time) discount factor is:

$$\phi_i(t) = \exp(-x_i t) \quad (14)$$

Weitzman considers  $x_i$  to be a random variable. For each individual,  $x_i$  is a random draw from a gamma distribution with scale parameter  $d$  and shape parameter  $c$ , both strictly positive and assumed to be constant across the population:

$$f(x) = \frac{b^c}{\Gamma(c)} x^{c-1} \exp(-bx), \quad b, c > 0 \quad (15)$$

The mean of this gamma distribution is  $\mu = c/b$  and the variance is  $\sigma^2 = c/b^2$ . In aggregating the diverse individual opinions about appropriate discount rates spanned by his sample, Weitzman proposes an aggregate  $\phi(t)$ , the “expected present discounted value of a dollar at time  $t$ ”, namely

$$\phi(t) \equiv \int_0^\infty \exp(-xt) f(x) dx. \quad (16)$$

Note that the expectation is the expected value of the continuous-time discounting function  $\exp(-xt)$ , rather than the expectation of discount rates  $x$  themselves. Weitzman further notes that the marginal or instantaneous effective discount rate at time  $t$  is defined to be

$$r(t) \equiv -\frac{\dot{\phi}(t)}{\phi(t)} \quad (17)$$

For the gamma probability density function, Weitzman shows that the expectation in (16)



can be expressed as

$$\phi(t) = \left( \frac{b}{b+t} \right)^c = \left( 1 + \frac{1}{b}t \right)^{-c} \quad (18)$$

Alternatively, if the expectation is expressed in terms of the mean  $\mu = c/b$  and the variance  $\sigma^2 = c/b^2$  of the gamma distribution, it begins to resemble the Loewenstein and Prelec (1992) generalized hyperbolic discounting formula. In terms of  $\mu$  and  $\sigma^2$ , the formula is:

$$\phi(t) = \left( 1 + \frac{\sigma^2}{\mu}t \right)^{-(\mu^2/\sigma^2)} \quad (19)$$

whereas, in terms of  $\beta$  and  $\gamma$ , it is:

$$\phi_g(t) = (1 + \gamma t)^{-\beta/\gamma} \quad (20)$$

If we let  $\beta = \mu = c/b$  and  $\gamma = \sigma^2/\mu = 1/b$ , it is clear that the two formulations are identical. Weitzman's formula for the expected present discounted value of a dollar at time  $t$  equals the generalized hyperbolic discounting formula.

In the one-parameter hyperbolic discounting function of Harvey (1986), explored recently by Keller and Strazzera (2002), the restriction  $\gamma = 1$  is imposed to yield a simplified discounting function:

$$\phi_a(t) = (1 + t)^{-\beta} \quad (21)$$

The  $\gamma$  parameter in the Loewenstein and Prelec (1992) formulation of the discounting formula corresponds to  $(1/b)$  in Weitzman's parameterization of the gamma distribution. The gamma distribution collapses to an exponential distribution when  $c = 1$ .

The conventional continuous-time discounting function,  $\phi(t) = \exp(-\beta t)$ , is a limiting case of the generalized hyperbolic discounting function when  $\gamma \rightarrow 0$ . This is easiest to

see if one considers the instantaneous effective discount rate at time  $t$  for the generalized hyperbolic discount function  $\phi(t)$ :

$$r(t) = -\frac{\dot{\phi}(t)}{\phi(t)} = \frac{\beta}{1 + \gamma t} \quad (22)$$

Recall that the parameter  $\gamma$  corresponds to the variance of the underlying gamma distribution, divided by its mean. As  $\gamma$  approaches zero for any mean,  $\beta$ , of the distribution (note that  $\beta > 0$  is required), this variance must approach zero and the probability density function  $f(x)$  approaches a discrete mass with probability one at the mean  $\beta$ , whereupon the expectation in equation (16) converges to  $\phi(t) = \exp(-\beta t)$ , the familiar continuous-time exponential discounting formula. This is consistent with  $r(t)$  in equation (22) approaching  $\beta$ .

## APPENDIX II

### Log-Likelihood Function

We assume  $\varepsilon \sim \text{logistic}(0, \kappa_i)$  where the dispersion parameter  $\kappa$  may be distinct (proportional) across contexts or for different types of respondents. Implicitly in all logit-based random utility models, the linear “slope” coefficients (here, the  $\mu$  and  $\beta$  vectors) can only be estimated up to a scale factor (i.e. relative to the implicit dispersion parameter of the error term), so researchers often proceed in terms of a normalized scale factor that is equal to one. If error variances differ across subsets of the data, then a factor of proportionality, relative to the error dispersion for the numeraire subset, may be estimated for other subsets. To ensure positive proportionality for the non-numeraire dispersion factors, they can be estimated as  $\kappa_i = \exp(\kappa_i^*)$

The probability formulas that are relevant, for each different number of levels in the answer options presented to respondents in each of our split samples, can now be defined for the ordered logit models used in estimation. If each subsample were to be used independently, there would be  $m - 1$  unknown threshold parameters to be estimated for each format. (We label our thresholds as  $\alpha_{jk}$ , where  $j$  denotes the number of answer categories and  $k$  denotes the threshold number, counting from the bottom, starting with zero.) However, with the pooled data from all four variants, the boundary between “YES” and “NO” will be normalized to zero, which means that  $\alpha_{20} = 0$  and  $\alpha_{41} = 0$  in the 2-level and 4-level cases, respectively. The locations of the remaining thresholds are freely estimated (without symmetry restrictions) but it should be the case that  $\alpha_{30} < 0$  and  $\alpha_{31} > 0$  for the 3-level cases, and  $\alpha_{40} < 0$  and  $\alpha_{42} > 0$  for the 4-level cases, and  $\alpha_{50} < 0$ ,  $\alpha_{51} < 0$ , and  $\alpha_{52} > 0$ ,  $\alpha_{53} > 0$  for the 5-level cases. In each of these cases, we will assess whether outcomes with the expected sign are captured within the interval estimate for each threshold parameter.

Note that we cannot allow different thresholds according to the number of response

categories, while simultaneously allowing error variances to differ only by the number of response categories. This leaves either the thresholds, or the error variances, underidentified. We do allow our error terms to differ systematically with an array of other variables which are not strictly redundant with a set of dummy variables for the number of response categories.

The different options for the functional form of  $\Delta V_i$  are itemized in the discussion of specifications in the body of this paper. The probability formulas for each type of response format are as follows:

For the 2-level (Yes / No) format:

$$\begin{aligned} P2Y_i &= \frac{1}{1 + \exp(\alpha_{20}/\kappa_i - \Delta V_i)} \\ P2N_i &= \frac{\exp(\alpha_{20}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{20}/\kappa_i - \Delta V_i)} \end{aligned} \quad (23)$$

For the 3-level (Yes / Not Sure / No) format:

$$\begin{aligned} P3Y_i &= \frac{1}{1 + \exp(\alpha_{31}/\kappa_i - \Delta V_i)} \\ P3NS_i &= \left( \frac{\exp(\alpha_{31}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{31}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{30}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{30}/\kappa_i - \Delta V_i)} \right) \\ P3N_i &= \frac{\exp(\alpha_{30}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{30}/\kappa_i - \Delta V_i)} \end{aligned} \quad (24)$$

For the 4-level (Definitely Yes / Probably Yes / Probably No / Definitely No) format:

$$\begin{aligned} P4DY_i &= \frac{1}{1 + \exp(\alpha_{42}/\kappa_i - \Delta V_i)} \\ P4PY_i &= \left( \frac{\exp(\alpha_{42}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{42}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{41}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{41}/\kappa_i - \Delta V_i)} \right) \\ P4PN_i &= \left( \frac{\exp(\alpha_{41}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{41}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{40}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{40}/\kappa_i - \Delta V_i)} \right) \\ P4DN_i &= \frac{\exp(\alpha_{40}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{40}/\kappa_i - \Delta V_i)} \end{aligned} \quad (25)$$

For the 5-level (Definitely Yes / Probably Yes / Not Sure / Probably No / Definitely No) format:

$$\begin{aligned}
P5DY_i &= \frac{1}{1 + \exp(\alpha_{53}/\kappa_i - \Delta V_i)} \\
P5PY_i &= \left( \frac{\exp(\alpha_{53}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{53}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{52}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{52}/\kappa_i - \Delta V_i)} \right) \\
P5NS_i &= \left( \frac{\exp(\alpha_{52}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{52}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{51}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{51}/\kappa_i - \Delta V_i)} \right) \\
P5PN_i &= \left( \frac{\exp(\alpha_{51}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{51}/\kappa_i - \Delta V_i)} \right) - \left( \frac{\exp(\alpha_{50}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{50}/\kappa_i - \Delta V_i)} \right) \\
P5DN_i &= \frac{\exp(\alpha_{50}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{50}/\kappa_i - \Delta V_i)}
\end{aligned} \tag{26}$$

Each threshold parameter  $\alpha_{jk}$  is normalized by the dispersion parameter,  $\kappa_i$ , for the error term in the relevant subsample of data.

The last necessary ingredient for the development of the log-likelihood function for this model is a set of indicators for choices. Indicators have the general format  $DnX_i$ . The value of  $n$  indicates how many answer levels were offered to the respondent ( $n = 2, 3, 4, 5$ ), and  $X$  includes  $Y$  and  $N$  for Yes and No, with  $P$  for the modifier “probably” and  $D$  for “definitely”.  $NS$  is the abbreviation for the “not sure” category. All indicators take a value of 1 if the designated response is selected, and are 0 otherwise.

All respondents provide either 3, 5, 7 or 13 responses to discounting questions. The different orderings and different formats of the answer options were randomized across split samples, so the log-likelihood formulas appropriate for each number of response options can simply be summed. The log-likelihood to be maximized by appropriate choices of the unknown

parameters can now be written in its most compact form as follows:

$$\begin{aligned}
\text{Log}L &= \sum_{i=1}^{ND2} [D2Y_i \ln(P2Y_i)] + D2N_i \ln(P2N_i) \\
&+ \sum_{i=1}^{ND3} [D3Y_i \ln(P3Y_i) + D3NS_i \ln(P3NS_i) + D3N_i \ln(P3N_i)] \\
&+ \sum_{i=1}^{ND4} [D4DY_i \ln(P4DY_i) + D4PY_i \ln(P4PY_i) \\
&\quad + D4PN_i \ln(P4PN_i) + D4DN_i \ln(P4DN_i)] \\
&+ \sum_{i=1}^{ND5} [D5DY_i \ln(P5DY_i) + D5PY_i \ln(P5PY_i) \\
&\quad + D5NS_i \ln(P5NS_i) + D5PN_i \ln(P5PN_i) \\
&\quad + D5DN_i \ln(P5DN_i)]
\end{aligned} \tag{27}$$

**APPENDIX III – (Available from the authors)**

Table III.1 - Age Distribution in the Sample

| Approximate Age |            | Frequency | Percent | Cum.   |
|-----------------|------------|-----------|---------|--------|
| ("midpoints")   | (range)    |           |         |        |
| 18              | 20 or less | 856       | 41.51   | 41.51  |
| 23              | 21-25      | 938       | 45.59   | 87.00  |
| 28              | 26-30      | 153       | 7.42    | 94.42  |
| 36              | 31-40      | 79        | 3.83    | 98.25  |
| 46              | 41-50      | 30        | 1.45    | 99.71  |
| 58              | 51-64      | 4         | 0.19    | 99.90  |
| 69              | 65 or more | 2         | 0.10    | 100.00 |
| Total           |            | 2062      | 100.00  |        |

Table III.2 - Income Distribution in the Sample

| Annual Family Income Now |                  | Frequency | Percent | Cum.   |
|--------------------------|------------------|-----------|---------|--------|
| ("midpoints")            | (range)          |           |         |        |
| 8                        | <\$10,000        | 101       | 4.90    | 4.90   |
| 15                       | \$10,000-20,000  | 157       | 7.61    | 12.51  |
| 25                       | \$20,000-30,000  | 206       | 9.99    | 22.50  |
| 40                       | \$30,000-50,000  | 361       | 17.51   | 40.01  |
| 62.5                     | \$50,000-75,000  | 405       | 19.64   | 59.65  |
| 87.5                     | \$75,000-100,000 | 371       | 17.99   | 77.64  |
| 125                      | >\$100,000       | 461       | 22.36   | 100.00 |
| Total                    |                  | 2062      | 100.00  |        |

Table III.3 – Distribution of Responses to question “Would you prefer this lump sum”?  
 (Individual respondents saw either 3, 5, 7 or 13 lump sums; lump sums for 30% and 50% implied discount rates were replaced by 7% and 9% implied rates)

Two-level answers:

---

| Implied <sup>a</sup><br>Disc. Rate |   | Yes   |   | No    |   | Row<br>Count |
|------------------------------------|---|-------|---|-------|---|--------------|
| 1%                                 | - | 373   | - | 41    | - | 414          |
| 2%                                 | - | 109   | - | 18    | - | 127          |
| 3%                                 | - | 332   | - | 80    | - | 412          |
| 4%                                 | - | 217   | - | 60    | - | 277          |
| 5%                                 | - | 184   | - | 80    | - | 264          |
| 6%                                 | - | 155   | - | 120   | - | 275          |
| 7%                                 | - | 159   | - | 158   | - | 317          |
| 8%                                 | - | 99    | - | 96    | - | 195          |
| 9%                                 | - | 125   | - | 193   | - | 318          |
| 10%                                | - | 79    | - | 159   | - | 238          |
| 12%                                | - | 99    | - | 201   | - | 300          |
| 15%                                | - | 46    | - | 120   | - | 166          |
| 20%                                | - | 87    | - | 288   | - | 375          |
| 30%                                | - | 4     | - | 25    | - | 29           |
| 50%                                | - | 14    | - | 83    | - | 97           |
| Total                              |   | 2,082 |   | 1,722 |   | 3,804        |

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Three-level answers:

---

| Implied<br>Disc. Rate |   | Yes   | Not Sure | No    |   | Row<br>Count |
|-----------------------|---|-------|----------|-------|---|--------------|
| 1%                    | - | 353   | 30       | 31    | - | 414          |
| 2%                    | - | 113   | 8        | 21    | - | 142          |
| 3%                    | - | 309   | 60       | 64    | - | 433          |
| 4%                    | - | 186   | 38       | 45    | - | 269          |
| 5%                    | - | 157   | 43       | 87    | - | 287          |
| 6%                    | - | 146   | 47       | 95    | - | 288          |
| 7%                    | - | 125   | 65       | 127   | - | 317          |
| 8%                    | - | 79    | 45       | 83    | - | 207          |
| 9%                    | - | 103   | 50       | 174   | - | 327          |
| 10%                   | - | 66    | 33       | 136   | - | 235          |
| 12%                   | - | 75    | 45       | 210   | - | 330          |
| 15%                   | - | 41    | 13       | 122   | - | 176          |
| 20%                   | - | 73    | 32       | 275   | - | 380          |
| 30%                   | - | 8     | 3        | 21    | - | 32           |
| 50%                   | - | 15    | 2        | 80    | - | 97           |
| Total                 |   | 1,849 | 514      | 1,571 |   | 3,934        |

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Four-level answers:

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| Implied<br>Disc. Rate | Def Yes | Prob Yes |   | Prob No | Def No | Row<br>Count |
|-----------------------|---------|----------|---|---------|--------|--------------|
| 1%                    | 283     | 61       | - | 25      | 15     | 384          |
| 2%                    | 81      | 31       | - | 12      | 6      | 130          |
| 3%                    | 209     | 120      | - | 38      | 25     | 392          |
| 4%                    | 111     | 82       | - | 39      | 27     | 259          |
| 5%                    | 96      | 78       | - | 47      | 34     | 255          |
| 6%                    | 66      | 83       | - | 73      | 45     | 267          |
| 7%                    | 77      | 87       | - | 80      | 56     | 300          |
| 8%                    | 42      | 47       | - | 49      | 48     | 186          |
| 9%                    | 55      | 59       | - | 99      | 95     | 308          |
| 10%                   | 38      | 41       | - | 63      | 86     | 228          |
| 12%                   | 45      | 43       | - | 75      | 123    | 286          |
| 15%                   | 22      | 23       | - | 43      | 73     | 161          |
| 20%                   | 46      | 42       | - | 68      | 197    | 353          |
| 30%                   | 4       | 2        | - | 4       | 18     | 28           |
| 50%                   | 9       | 7        | - | 14      | 54     | 84           |
| Total                 | 1,184   | 806      |   | 729     | 902    | 3,621        |

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Five-level answers:

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| Implied<br>Disc. Rate | Def Yes | Prob Yes | Not Sure | Prob No | Def No | Row<br>Count |
|-----------------------|---------|----------|----------|---------|--------|--------------|
| 1%                    | 325     | 71       | 28       | 15      | 13     | 452          |
| 2%                    | 107     | 33       | 4        | 9       | 8      | 161          |
| 3%                    | 246     | 121      | 42       | 35      | 28     | 472          |
| 4%                    | 105     | 105      | 44       | 24      | 18     | 296          |
| 5%                    | 112     | 94       | 40       | 36      | 35     | 317          |
| 6%                    | 86      | 64       | 61       | 57      | 48     | 316          |
| 7%                    | 78      | 65       | 86       | 64      | 60     | 353          |
| 8%                    | 51      | 39       | 42       | 42      | 48     | 222          |
| 9%                    | 68      | 47       | 65       | 90      | 98     | 368          |
| 10%                   | 41      | 37       | 30       | 74      | 84     | 266          |
| 12%                   | 51      | 33       | 44       | 98      | 126    | 352          |
| 15%                   | 20      | 14       | 20       | 43      | 94     | 191          |
| 20%                   | 49      | 21       | 38       | 92      | 222    | 422          |
| 30%                   | 1       | 0        | 1        | 11      | 25     | 38           |
| 50%                   | 7       | 2        | 3        | 11      | 76     | 99           |
| Total                 | 1,347   | 746      | 548      | 701     | 983    | 4,325        |

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<sup>a</sup> Lump sums and annual amounts varied widely. Displayed lump sums corresponded to the exponentially discounted present value of the specified stream of payments for the specified number of years, but were rounded to two significant figures if less than \$10,000 and to three significant figures if greater than \$10,000. The discount rates used in this table are the approximate exponential discount rates embodied in the rounded lump sum. Rounding was necessary to reduce the complexity of the choice task for respondents.