

246 Midterm 2 Study Guide

Midterm 2 will cover computations of derivatives, including using the definition of the derivative, knowing the derivatives of elementary functions, knowing the standard rules for computing derivatives of functions built from elementary functions and techniques involving implicit differentiation.

You will need to know or be able to come up with:

1. ... the following derivatives: x^r , a^x , $\sin(x)$, $\cos(x)$, $\log_a(x)$, $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$.
2. ... the derivative of a sum, product, quotient or composition of functions.
3. ... the derivative of the inverse to a function and the derivative of a function with x in both the base and exponent.
4. ... the equation of the tangent line to a function at a given input x .
5. ... the slope of the tangent line to an equation that's difficult to solve for y at a point (x, y) .

Some problems (to supplement your homework and problems from the book):

1. $\frac{d}{dx} x^3 + 3x - 1$
2. $\frac{d}{dx} \sin^{-1}(x)$
3. $\frac{d}{dx} 2(x-1)^{(x+3)}$
4. $\frac{d}{dx} \sin(x) \sin^{-1}(x)$
5. $\frac{d}{dx} \sin(\sin^{-1}(x))$
6. $\frac{d}{dx} \ln(2x^2 + \tan(x))$
7. $\frac{d}{dx} (\ln(x))^{\ln(x)}$
8. $\frac{d}{dx} \frac{x^2+3x-1}{\sin(x)\cos(x)}$
9. $\frac{d}{dx} \sin^2(x) - \cos^2(x)$
10. $\frac{d}{dx} \tan^2(x)$
11. $\frac{d^2}{(dx)^2} \sqrt{\sqrt{2x-1}-1}$
12. Use the definition of derivative to compute the derivative of $f(x) = x^2$ and $g(x) = \frac{1}{x}$.

13. Find the slope of the tangent line to each of the above at $x = 1$, if it exists. If not, note this.
14. Find the equation of the tangent line to each of the above at $x = 1$, if it exists. If not, note this.
15. Find the slope of the tangent line of $y^3 + 4x^2y - 4 = 0$ at $(1, 1)$, $\frac{x}{y+x}$ at $(0, 0)$ and $\sin(xy) = 1$ at $(\pi/2, 1)$,
16. Find the derivative of $\log_a(x)$ or $\sqrt[n]{x}$ using implicit differentiation – don't just write down the answer.
17. I am filling a spherical balloon with air at a rate of $1 \text{ m}^3/\text{minute}$. Assuming that the balloon starts empty, how fast is its radius increasing after 5 minutes?