43. **General solution of the wave equation**

Consider a one-dimensional wave equation

\[
(\partial_t^2 - c^2 \partial_x^2) f(x,t) = 0
\]

Show that the general solution constructed by Fourier transform in Ch. 4 §2.2 has the form of the d’Alembert solution from ch. 4 §1.2, and vice versa. 

(2 points)

44. **Liénard-Wiechert potentials**

Consider a point charge \( e \) that moves on a given trajectory \( \mathbf{X}(t) \) with velocity \( \mathbf{v}(t) = \dot{\mathbf{X}}(t) \) which results in charge and current densities

\[
\rho(x,t) = e \delta(x - \mathbf{X}(t)), \quad j(x,t) = e \mathbf{v}(t) \delta(x - \mathbf{X}(t))
\]

Show that the resulting retarded potentials have the form

\[
\varphi(x,t) = \frac{e}{|x - \mathbf{X}(t_-)| - \mathbf{v}(t_-) \cdot (x - \mathbf{X}(t_-))} \quad A(x,t) = \frac{1}{c} \mathbf{v}(t_-) \varphi(x,t)
\]

where \( t_- \) is the solution of

\[
t_- = t - \frac{1}{c} |x - \mathbf{X}(t_-)| \quad (*)
\]

These are known as Liénard-Wiechert potentials after Alfred-Marie Liénard and Emil Wiechert, who derived them in 1898 and 1900, respectively.

*hint*: Show that the equation (*) for \( t_- \) has one and only one solution, and use the properties of the \( \delta \)-function discussed in ch. 3 §2.5.

(6 points)

45. **Potential of a uniformly moving charge**

Consider a charge \( e \) moving uniformly along the \( x \)-axis with velocity \( \mathbf{v} \): \( \mathbf{X}(t) = (vt, 0, 0) \). Determine the Liénard-Wiechert potentials explicitly, and show that the result is that same as the one obtained in ch. 3 §3.4 by means of a Lorentz transformation.

(6 points)

46. **Wave equations for the electromagnetic fields**

Show directly from the Maxwell equations, without introducing potentials, that the fields obey the inhomogeneous wave equations

\[
\Box E = -4\pi \left( \nabla \rho + \frac{1}{c^2} \partial_t^2 j \right), \quad \Box B = \frac{4\pi}{c} \nabla \times j.
\]

(2 points)