1. **Russell’s Paradox**

a) Consider the set $M$ defined as the set of all sets that do not contain themselves as an element: $M = \{ x; x \notin x \}$. Discuss why this is a problematic definition.

b) A less abstract version of Russell’s paradox is known as the barber’s paradox: Consider a town where all men either shave themselves, or let the barber shave them and don’t shave themselves. Now consider the statement

*The barber is a man in town who shaves all men who do not shave themselves, and only those.*

Discuss why this definition of the barber is problematic (assuming there actually is a barber in town).

*hint: Ask “Does the barber shave himself?”*

c) Suppose the definition of the barber is modified to read

*The barber shaves all men in town who do not shave themselves, and only those.*

Discuss what this modification does to the paradox.

(3 points)

2. **Distributive property of the union and intersection relations**

Convince yourself graphically of the distributive property of the relations $\cup$ and $\cap$, ch.1 §1.1 remark (9)(iii). That is, for any three sets $A$, $B$, $C$,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(2 points)

3. **Mappings**

Are the following $f : X \to Y$ true mappings? If so, are they surjective, or injective, or both?

a) $X = Y = \mathbb{Z}$, $f(m) = m^2 + 1$.

b) $X = Y = \mathbb{N}$, $f(n) = n + 1$.

c) $X = \mathbb{Z}$, $Y = \mathbb{R}$, $f(x) = \log x$.

d) $X = Y = \mathbb{R}$, $f(x) = e^x$.

(2 points)

4. **Parabolic Mapping**

Consider $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = an^2 + bn + c$, with $a, b, c \in \mathbb{Z}$.

a) For which triplets $(a, b, c)$ is $f$ surjective?

b) For which $(a, b, c)$ is $f$ injective?

(4 points)