34. **Spherical waves**

Consider the wave equation

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) f(x, t) = 0
\]

Find and discuss the most general solution that has the form

\[ f(x, t) = u(r, t)/r \]

where \( r = |x| \).

(3 points)

35. **Cosmological redshift**

Edwin Hubble observed the following relation between the wavelength of spectral lines in galaxies and the distance of the galaxies from the earth:

\[
\frac{\lambda - \lambda_0}{\lambda} = \frac{Hr}{c}
\]

where \( \lambda \) is the wavelength of a spectral line as observed in the galaxy, \( \lambda_0 \) is the wavelength of the same spectral line as measure in the laboratory, \( r \) is the distance of the galaxy, and \( c \) is the speed of light. \( H \) is observed to be roughly \( H \approx 68 \text{ (km/s)/Mpc} \) (1 Mpc = 3.26 \times 10^6 \text{ light years}).

a) Assuming that the observed red shift is due to the nonrelativistic Doppler effect, and that the motion of the galaxy is purely radial, find a relation between the distance of a galaxy and its velocity with respect to the earth.

b) How long did it take a galaxy that’s now at distance \( r \) to get there? Use the result to estimate the age of the universe.

c) Hubble’s original estimate was \( H \approx 530 \text{ (km/s)/Mpc} \). Why does this value pose a problem?

(3 points)

36. **General solution of the wave equation**

Consider a one-dimensional wave equation

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} \right) f(x, t) = 0
\]

Show that the general solution constructed by Fourier transform in Ch. 4 §2.2 has the form of the d’Alembert solution from ch. 4 §1.2, and vice versa.

(2 points)

37. **Wave equations for the electromagnetic fields**

Show directly from the Maxwell equations, without introducing potentials, that the fields obey the inhomogeneous wave equations

\[
\Box E = -4\pi \left( \nabla \rho + \frac{1}{c^2} \partial_t j \right), \quad \Box B = \frac{4\pi}{c} \nabla \times j.
\]

(2 points)