14. **Particle in homogeneous \( E \) and \( B \) fields**

Consider a point particle (mass \( m \), charge \( e \)) in homogeneous fields \( \mathbf{B} = (0, 0, B) \) and \( \mathbf{E} = (0, E_y, E_z) \). Treat the motion of the particle nonrelativistically.

a) Show that the motion in \( z \)-direction decouples from the motion in the \( x-y \) plane, and find \( z(t) \).

b) Consider \( \xi := x + iy \). Find the equation of motion for \( \xi \), and its most general solution.

*hint: Define the cyclotron frequency \( \omega = eB/mc \), and remember how to solve inhomogeneous ODEs.*

c) Show that the time-averaged velocity perpendicular to the plane defined by \( \mathbf{B} \) and \( \mathbf{E} \) is given by the *drift velocity*

\[
\langle v \rangle = c \mathbf{E} \times \mathbf{B}/B^2
\]

Show that \( E_y/B \ll 1 \) is necessary and sufficient for the non relativistic approximation to be valid.

d) Show that the path projected onto the \( x-y \) plane can have three qualitatively different shapes, and plot a representative example for each.

(6 points)

15. **Harmonic oscillator coupled to a magnetic field**

Consider a charged 3-d classical harmonic oscillator (oscillator frequency \( \omega_0 \), charge \( e \)). Put the oscillator in a homogeneous time-independent magnetic field \( \mathbf{B} = (0, 0, B) \). Show that the motion remains oscillatory, and find the oscillation frequencies in the directions parallel and perpendicular, respectively, to \( \mathbf{B} \).

(4 points)

16. **Relativistic motion in parallel electric and magnetic fields**

Consider a relativistic charged particle (mass \( m \), charge \( e \)) in parallel homogeneous electric and magnetic fields \( \mathbf{E} = (0, 0, E), \mathbf{B} = (0, 0, B) \).

a) Show that the equation of motion for the \( z \)-component of the momentum \( p_z \) decouples from \( p_x \) and \( p_y \), and that the momentum perpendicular to the \( z \)-axis is a constant of motion: \( p_x^2 + p_y^2 = p_{\perp}^2 = \text{const} \).

b) Choose the zero of time such that \( p_z(t = 0) = 0 \), and show that with a suitable chosen origin the \( z \)-component of the particle’s position can be written

\[
z(t) = \frac{1}{eE} \sqrt{\frac{T_0^2}{c^2} + c^2e^2E^2t^2}
\]

where \( T_0 \) is the kinetic energy (i.e., the energy of the particle without the potential energy due to the fields) at time \( t = 0 \).

*hint: If you have trouble, recall Einstein’s law of falling bodies from PHYS 611. You can find my version at http://pages.uoregon.edu/dbelitz/teaching/2013_14/PHYS_611-4/ , Assignment # 5, Problem 21.*
c) Introduce a parameter \( \varphi \) via \( d\varphi/dt = ceB/T(t) \), with \( T(t) \) the time-dependent kinetic energy. Show that the orbit of the particle can be represented in the parametric form

\[
x = \frac{cp_\perp}{eB} \sin \varphi, \quad y = \frac{cp_\perp}{eB} \cos \varphi, \quad z = \frac{T_0}{eE} \cosh(\frac{E\varphi}{B})
\]

and explicitly find the relation between \( \varphi \) and \( t \).

*hint:* Consider \( \pi := p_x + ip_y \) and note that \( |\pi| = p_\perp = \text{const.} \) by the result of part a).

d) Describe and visualize the orbit, and discuss the motion in the limits of large and small times.

(14 points)