Problem Assignment # 5 10/24/2018
due 10/31/2018

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17. $\mathbb{R}$ **as a metric space**

Consider the reals $\mathbb{R}$ with $\rho : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by $\rho(x, y) = |x - y|$. Show that this definition makes $\mathbb{R}$ a metric space.

(3 points)

18. **Limits of sequences**

a) Show that a sequence in a metric space has at most one limit.

hint: Assume there are two limits, and use the triangle inequality to show that they must be the same.

b) Show that every sequency with a limit is a Cauchy sequence.

(3 points)

19. **Banach space**

Prove Proposition 1 from §4.6, i.e., show that the norm on the dual space B* of a Banach space B as defined in §4.6 def. 4 is a norm in the sense of the norm $||\ldots||$ defined on B itself in §4.6 def. 1.

(3 points)

20. **Hilbert space**

a) Show that the norm on a Hilbert space defined by §4.7 def. 1 is a norm in the sense of §4.6 def. 1.

hint: Use the Cauchy-Schwarz inequality (§4.7 lemma).

b) Show that the mappings ℓ defined in §4.7 def. 4 are linear forms in the sense of §4.3 def. 1(a).

(3 points)