PHYS 610 — Mathematical Methods for Scientists — F 2018

Problem Assignments # 7 — 11/07/2018
due 11/14/2018

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25. Transformations of tensor fields

a) Consider a covariant rank-n tensor field $t_{i_1 \ldots i_n}(x)$ and find its transformation law under normal coordinate transformations that is analogous to §5.1 def.1; i.e., find how $\tilde{t}_{i_1 \ldots i_n}(\tilde{x})$ is related to $t_{i_1 \ldots i_n}(x)$.

b) Convince yourself that your result is consistent with the transformation properties of (i) a covector x_i (the case $n = 1$), and (ii) the covariant components of the metric tensor g_{ij}.

(4 points)

26. Curl and divergence

Show that the curl and the divergence of a vector field transform as a pseudovector field and a scalar field, respectively.

(3 points)

27. Tensor products, and tensor traces

Prove Propositions 1 and 2 from ch. 1 §5.3.

(4 points)

28. Lindhard function

Consider the function $f : \mathbb{C} \to \mathbb{C}$ (which plays an important role in the theory of many-electron systems) defined by

$$f(z) = \log\left(\frac{z-1}{z+1}\right)$$

The *spectrum* $f'' : \mathbb{R} \to \mathbb{R}$ and the *reactive part* $f' : \mathbb{R} \to \mathbb{R}$ of f are defined by

$$f''(\omega) := \frac{1}{2i}\left[f(\omega + i0) - f(\omega - i0)\right] \qquad , \qquad f'(\omega) := \frac{1}{2}\left[f(\omega + i0) + f(\omega - i0)\right]$$

a) Show that f' and f'' are indeed real-valued functions.

b) Determine f'' and f' explicitly, and plot them for $-3 < \omega < 3$.

c) Show that

$$\int_{\infty}^{\infty} \frac{d\omega}{\pi}\, \frac{f''(\omega)}{\omega - z} = f(z)$$

(5 points)