PHYS 622 Classical Electrodynamics W 2019

Problem Assignment # 2 01/16/2019
due 01/23/2019

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5. **Energy-momentum tensor**

Consider the electromagnetic field in the absence of matter.

a) Show that the tensor field

$$H_\mu{}^\nu(x) = (\partial_\mu A_\alpha(x))\, \frac{\partial \mathcal{L}}{\partial\,(\partial_\nu A_\alpha(x))} - \delta_\mu{}^\nu\, \mathcal{L}$$

obeys the continuity equation

$$\partial_\nu\, H_\mu{}^\nu(x) = 0 \quad (*)$$

*note:* Notice that $H_\mu{}^\nu(x)$ is a generalization of Jacobi's integral in Classical Mechanics.

b) Show that (*) also holds for

$$\tilde{T}_\mu{}^\nu = H_\mu{}^\nu + \partial_\alpha \psi_\mu{}^{\nu\alpha}$$

where $\psi_\mu{}^{\nu\alpha}$ is any tensor field that is antisymmetric in the second and third indices, $\psi_\mu{}^{\nu\alpha}(x) = -\psi_\mu{}^{\alpha\nu}(x)$.

c) Show that $\psi_\mu{}^{\nu\alpha}$ can be chosen such that $\tilde{T}_\mu{}^\nu(x) = T_\mu{}^\nu(x)$, which provides an alternative proof that $T_\mu{}^\nu(x)$ obeys (*).

(5 points)

6. **Energy-momentum conservation in the presence of matter**

Prove the corollary of ch. 1 §2.3: In the presence of matter, the energy-momentum tensor obeys the continuity equation

$$\partial_\nu T_\mu{}^\nu(x) = \frac{-1}{c}\, F_\mu{}^\nu(x)\, J_\nu(x)$$

(2 points)

7. **Energy-momentum tensor for a massive scalar field**

Consider the massive scalar field from Problem 3:

$$\mathcal{L} = \frac{1}{2}\, (\partial_\mu \varphi)\, (\partial^\mu \varphi) - \frac{m^2}{2}\, \varphi^2$$

and the tensor field $H_\mu{}^\nu$ defined analogously to Problem 5:

$$H_\mu{}^\nu = (\partial_\mu \varphi)\, \frac{\partial \mathcal{L}}{\partial\,(\partial_\nu \varphi)} - \delta_\mu{}^\nu\, \mathcal{L}$$

Determine $H_\mu{}^\nu$ explicitly and show that

$$\partial_\nu H_\mu{}^\nu = 0$$

*hint:* Use the Euler-Lagrange equation determined in Problem 3a).

(3 points)

**.../over**

8. **Coulomb gauge**

Consider the 4-vector potential $A^\mu(x) = (\varphi(x), \boldsymbol{A}(x))$. Show that one can always find a gauge transformation such that

$$\boldsymbol{\nabla} \cdot \boldsymbol{A}(x) = 0$$

This choice is called *Coulomb gauge*.

(2 points)