PHYS 622 Classical Electrodynamics W 2019

Problem Assignment # 3

01/23/2019
due 01/30/2019

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9. **Particle in homogeneous *E* and *B* fields**

Consider a point particle (mass m, charge e) in homogeneous fields $\boldsymbol{B} = (0, 0, B)$ and $\boldsymbol{E} = (0, E_y, E_z)$. Treat the motion of the particle nonrelativistically.

a) Show that the motion in z-direction decouples from the motion in the x-y plane, and find $z(t)$.

b) Consider $\xi := x + iy$. Find the equation of motion for ξ, and its most general solution.

hint: Define the *cyclotron frequency* $\omega = eB/mc$, and remember how to solve inhomogeneous ODEs.

c) Show that the time-averaged velocity perpendicular to the plane defined by $\boldsymbol{B}$ and $\boldsymbol{E}$ is given by the *drift velocity*

$$\langle \boldsymbol{v} \rangle = c\, \boldsymbol{E} \times \boldsymbol{B}/\boldsymbol{B}^2$$

Show that $E_y/B \ll 1$ is necessary and sufficient for the non relativistic approximation to be valid.

d) Show that the path projected onto the x-y plane can have three qualitatively different shapes, and plot a representative example for each.

(6 points)

10. **Harmonic oscillator coupled to a magnetic field**

Consider a charged 3-d classical harmonic oscillator (oscillator frequency ω_0, charge e). Put the oscillator in a homogeneous time-independent magnetic field $\boldsymbol{B} = (0, 0, B)$. Show that the motion remains oscillatory, and find the oscillation frequencies in the directions parallel and perpendicular, respectively, to $\boldsymbol{B}$.

(4 points)

11. **Relativistic motion in parallel electric and magnetic fields**

Consider a relativistic charged particle (mass m, charge e) in parallel homogeneous electric and magnetic fields $\boldsymbol{E} = (0, 0, E)$, $\boldsymbol{B} = (0, 0, B)$.

a) Show that the equation of motion for the z-component of the momentum p_z decouples from p_x and p_y, and that the momentum perpendicular to the z-axis is a constant of motion: $p_x^2 + p_y^2 \equiv p_\perp^2 = \text{const.}$

b) Choose the zero of time such that $p_z(t = 0) = 0$, and show that with a suitable chosen origin the z-component of the particle's position can be written

$$z(t) = \frac{1}{eE} \sqrt{T_0^2 + c^2 e^2 E^2 t^2}$$

where T_0 is the kinetic energy (i.e., the energy of the particle without the potential energy due to the fields) at time $t = 0$.

hint: If you have trouble, recall Einstein's law of falling bodies from PHYS 611. You can find my version at http://pages.uoregon.edu/dbelitz/teaching/2013_14/PHYS_611-4/ , Assignment # 5, Problem 21.

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c) Introduce a parameter φ via $d\varphi/dt = ceB/T(t)$, with $T(t)$ the time-dependent kinetic energy. Show that the orbit of the particle can be represented in the parametric form

$$x = \frac{cp_\perp}{eB}\sin\varphi \quad , \quad y = \frac{cp_\perp}{eB}\cos\varphi \quad , \quad z = \frac{T_0}{eE}\cosh(E\varphi/B)$$

and explicitly find the relation between φ and t.

hint: Consider $\pi := p_x + ip_y$ and note that $|\pi| = p_\perp =$ const. by the result of part a).

d) Describe and visualize the orbit, and discuss the motion in the limits of large and small times.

(14 points)