PHYS 622 Classical Electrodynamics W 2019

Problem Assignment # 5 02/06/2019
due 02/13/2019

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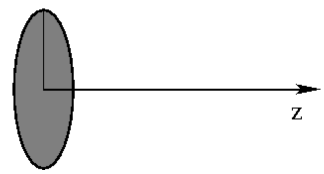
16. **Planar charge distributions**

a) Consider a homogeneously charged infinitesimally thin ring with radius R and total charge Q that is oriented perpendicular to the z-axis. Calculate the electric field on the z-axis.

b) The same for a homogeneously charged disk with charge density σ and radius R. Consider the limits $z \to \infty$, $z \to 0$, and $R \to \infty$, and ascertain that they makes sense.



(4 points)

17. **Spherically symmetric charge distributions**

Consider a spherically symmetric static charge distribution (in spherical coordinates): $\rho(\boldsymbol{x}) = \rho(r)$.

a) Express the electric field in terms of a one-dimensional integral over $\rho(r)$, and the electrostatic potential by a one-dimensional integral over the field.

hint: Make an *ansatz* for a purely radial field, $\boldsymbol{E}(\boldsymbol{x}) = E(r)\,\hat{e}_r$, and integrate Gauss's law over a spherical volume.

Explicitly calculate and plot the field $\boldsymbol{E}(\boldsymbol{x})$ and the potential $\varphi(\boldsymbol{x})$ for

b) a homogeneously charged sphere

$$\rho(\boldsymbol{x}) = \begin{cases} \rho_0 & \text{if } \; r \le r_0 \\ 0 & \text{if } \; r > r_0 \; . \end{cases}$$

c) a homogeneously charged spherical shell

$$\rho(\boldsymbol{x}) = \sigma_0 \, \delta(r - r_0) \; .$$

(8 points)

.../over

18. **Electrostatics in d dimensions (to be continued next week)**

Consider the third Maxwell equation in d dimensions:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{x}) = S_d\, \rho(\boldsymbol{x})$$

with the electric field $\boldsymbol{E}$ a d-vector, and S_d the area of the $(d-1)$-sphere: $S_{2n} = 2\pi^n/(n-1)!$ and $S_{2n+1} = 2^{2n+1} n! \pi^n/(2n)!$ for even and odd dimensions, respectively. Define a scalar potential $\varphi(\boldsymbol{x})$ in analogy to the $3-d$ case, such that

$$\boldsymbol{E}(\boldsymbol{x}) = -\boldsymbol{\nabla}\varphi(\boldsymbol{x})$$

and consider Poisson's equation

$$\boldsymbol{\nabla}^2 \varphi(\boldsymbol{x}) = -S_d\, \rho(\boldsymbol{x})$$

note: Here we consider a generalization of electrostatics to d-dimensional space, NOT a d-dimensional charge distribution embedded in 3-dimensional space.

a) Show that the Green function $G_d(\boldsymbol{x})$ function for Poisson's equation, i.e., the solution of

$$\boldsymbol{\nabla}^2 G_d(\boldsymbol{x}) = -S_d\, \delta(\boldsymbol{x})$$

is given by

$$G_d(\boldsymbol{x}) = \frac{1}{d-2} \frac{1}{|\boldsymbol{x}|^{d-2}}$$

for all $d \neq 2$, and by

$$G_2(\boldsymbol{x}) = \ln(1/|\boldsymbol{x}|)$$

for $d = 2$.

hint: For $d = 1$, differentiate directly, using PHYS 610 Problem 36b). For $d \geq 2$, show that $G_d(\boldsymbol{x})$ is a harmonic function for all $\boldsymbol{x} \neq 0$, then integrate $\boldsymbol{\nabla}^2 G_d$ over a hypersphere around the origin and use Gauss's law.

(4 points)