PHYS 622 Classical Electrodynamics W 2019

Problem Assignment # 9 03/06/2019
due 03/13/2019

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37. **Potentials in Coulomb gauge**

Consider the potentials φ and $\boldsymbol{A}$ in the Coulomb gauge, i.e., the field equations from ch.4 §1.2 proposition 2. Show explicitly that the resulting asymptotic electric and magnetic fields are the same as those calculated in the Lorenz gauge in ch.4 §3.

hint: Show that the scalar potential does not contribute to the electric field, and show that the asymptotic vector potential now reads

$$\boldsymbol{A}(\boldsymbol{x}, t) = -\hat{\boldsymbol{x}} \times \left[\hat{\boldsymbol{x}} \times \frac{1}{rc} \int d\boldsymbol{y}\, \boldsymbol{j}(\boldsymbol{y}, t_r) \right]$$

instead of the expression derived in ch. 4 §3.1. Then calculate the fields.

(8 points)

38. **Radiation from cyclotron motion**

Consider a point mass m with charge e that moves in a plane perpendicular to a homogeneous magnetic field $\boldsymbol{B}$. Assume nonrelativistic motion, $v \ll c$

a) Find the power radiated by the particle.

b) Show that the energy of the particle decreases with time according to $E(t) = E_0\, e^{-t/\tau}$, and determine the timescale τ.

c) Find τ in seconds for an electron in a magnetic field of 1 Tesla.

(4 points)

39. **Radiating harmonic oscillator**

Consider particle with charge e and mass m in a one-dimensional harmonic potential. Let the frequency of the harmonic oscillator by ω_0.

a) Find the power radiated by the particle, averaged over one oscillation period, as a function of the energy E of the oscillator.

   hint: Remember the virial theorem, which for a harmonic potential says $\bar{V} = \bar{T} = E/2$, with V, T, and E the potential, kinetic, and total energy, respectively, of the particle, and the bar denoting a time average.

b) Show that the energy of the oscillator again decreases exponentially, $E(t) = E_0\, e^{-t/\tau}$.

c) Determine τ in seconds for e and m the electron charge and mass, respectively, and $\omega_0 = 10^{15}\,\text{sec}^{-1}$ (a typical atomic frequency).

(4 points)

41. **Absence of dipole radiation**

Show that a system of particles that all have to the same ratio of charge to mass and are not subject to any external forces cannot emit either electric or magnetic dipole radiation.

(3 points)