PHYS 610 Mathematical Methods for Scientists F 2020

Final Exam 12/06 - 09/2020

This is a closed-book exam. No notes, books, online resources, Mathematica, etc., please.

You can choose any contiguous two-hour period you like for working on the exam, but once you open it you must finish it within the next two hours. When you are done, scan it and email it to dbelitz@uoregon.edu. Time needed for reading these instructions, and for scanning and mailing, do not count towards the two hours. The exam is due no later than Wednesday, Dec. 9, 5pm PST. Since different people will be working on the exam at different times I will not answer any questions, neither privately nor publicly. If you feel there is something wrong or ambiguous about the formulation of a problem, clearly state your assumptions and keep going.

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1. **Complex integration**

note: If, in any part of this problem, parts of the contour you wish to integrate over obviously do not contribute, say so and give a simple argument for why that's true. You don't have to prove that it's true. Similarly, if you feel that the analytic structure of a function is obvious, you can just state what it is without proof.

a) Consider the complex function

$$f(z) = \frac{1}{1+z^2}$$

Where is f analytic? Classify the singularities of f, if any. Find the residues of the poles, if any.

b) Construct the Laurent series for f up to and including the constant term in the vicinity of the poles and verify the values of the residues you found in part a).

c) Repeat part a) for the function

$$g(z) = \frac{1}{(1+z^2)^2}$$

d) Use the residue theorem to evaluate the integral

$$I = \int_{-\infty}^{\infty} dx\, \frac{1}{1+x^2}$$

Show explicitly that you obtain the same result irrespective of whether you close the contour in the upper or the lower half plane.

e) Evaluate the integral

$$J = \int_{-\infty}^{\infty} dx\, \frac{\sin(x+a)}{1+x^2} \qquad (a \in \mathbb{R})$$

f) Evaluate the integral

$$K = \int_{-\infty}^{\infty} dx\, \frac{1}{(1+x^2)^2}$$

(13 points)

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2. **Minkowski tensors**

Let F be an antisymmetric rank-2 tensor in the Minkowski space M_4 with metric $g = (+, -, -, -)$.

a) How many independent components does F have? Show that the independent components can be expressed in terms of a vector and a pseudovector, respectively, in the Euclidian subspace E_3 of M_4.

   note: Just give simple arguments, no formal proofs are necessary.

b) Express the covariant components $F_{\mu\nu}$, and the mixed components $F^{\mu}{}_{\nu}$ and $F_{\mu}{}^{\nu}$, in terms of the contravariant components $F^{\mu\nu}$.

c) What are the transformation properties of the object $I = F^{\mu\nu}F_{\mu\nu}$ under normal coordinate transformations? What does this imply for the moduli of the Euclidian vector and pseudovector from part a) while transforming from one normal coordinate system to another?

(8 points)

3. **Finite sum**

Show that

$$\sum_{k=0}^{n}(2k+1)^2 = \frac{1}{3}(1+n)(1+2n)(3+2n)$$

for all $n \in \mathbb{N}$.

(5 points)