PHYS 610 Mathematical Methods for Scientists F 2020

Problem Assignment # 3 10/15/2020
due 10/22/2020

1.2.2 **Products**

Prove the corollary to proposition 2 of ch.1 §2.2: If a is an element of a multiplicative group, and $n, m \in \mathbb{N}$, then

a) $a^n\, a^m = a^{n+m}$

b) $(a^n)^m = a^{nm}$

(2 points)

1.2.3 **The group** S_3

a) Compile the group table for the symmetric group S_3. Is S_3 abelian?

b) Find all subgroups of S_3. Which of these are abelian?

(6 points)

1.2.4 **Abelian groups**

Let $(G, \vee)$ be a group with neutral element e. Let $a \in G$ be a fixed element, and define a mapping $\varphi : G \to G$ by $\varphi(x) = a \vee x \vee a^{-1}\ \forall x \in G$.

a) Show that φ defines an automorphism on G, called an *inner automorphism*.

b) Show that abelian groups have no inner automorphisms except for the identity mapping $\varphi(x) = x$.

c) Let $g \vee g = e\ \forall g \in G$. Prove that G is abelian.

(6 points)

1.3.1 **Fields**

a) Show that the set of rational numbers $\mathbb{Q}$ forms a commutative field under the ordinary addition and multiplication of numbers.

b) Consider a set F with two elements, $F = \{\theta, e\}$. On F, define an operation "plus" $(+)$, about which we assume nothing but the defining properties

$$\theta + \theta = \theta \quad , \quad \theta + e = e + \theta = e \quad , \quad e + e = \theta$$

Further, define a second operation "times" $(\cdot)$, about which we assume nothing but the defining properties

$$\theta \cdot \theta = e \cdot \theta = \theta \cdot e = \theta \quad , \quad e \cdot e = e$$

Show that with these definitions (and **no** additional assumptions), F is a field.

(7 points)