PHYS 610 Mathematical Methods for Scientists F 2020

Problem Assignment # 4 10/22/2020
due 10/29/2020

1.4.1. **Function space**

Consider the set C of continuous functions $f : [0,1] \to \mathbb{R}$. Show that by suitably defining an addition on C, and a multiplication with real numbers, one can make C an additive vector space over $\mathbb{R}$.

(2 points)

1.4.2. **The space of rank-2 tensors**

a) Prove the theorem of ch.1 §4.3: Let V be a vector space V of dimension n over K. Then the space of rank-2 tensors, defined via bilinear forms $f : V \times V \to K$, forms a vector space of dimension n^2.

b) Consider the space of bilinear forms f on V that is equivalent to the space of rank-2 tensors, and construct a basis of that space.

hint: On the space of tensors, define a suitable addition and multiplication with scalars, and construct a basis of the resulting vector space.

(5 points)

1.4.3. **Cross product of 3-vectors**

Let $x, y \in \mathbb{R}_3$ be vectors, and let ϵ_{ijk} be the Levi-Civita symbol. Show that the (covariant) components of the cross product $x \times y$ are given by

$$(x \times y)_i = \epsilon_{ijk} x^j y^k$$

(1 point)

1.4.4. **Symmetric tensors**

Let V be an n-dimensional vector space over K with some basis, let $f : V \times V \to K$ be a bilinear form, and let t be the rank-2 tensor defined by f. Show that f is symmetric, i.e. $f(x,y) = f(y,x)\ \forall x, y \in V$, if and only if the components of the tensor with respect to the given basis are symmetric, i.e., $t_{ij} = t_{ji}$.

(2 points)