PHYS 610 Mathematical Methods for Scientists F 2020

Problem Assignments # 7 11/12/2020
due 11/19/2020

1.5.1. **Transformations of tensor fields**

a) Consider a covariant rank-n tensor field $t_{i_1 \ldots i_n}(x)$ and find its transformation law under normal coordinate transformations that is analogous to §5.1 def.1; i.e., find how $\tilde{t}_{i_1 \ldots i_n}(\tilde{x})$ is related to $t_{i_1 \ldots i_n}(x)$.

b) Convince yourself that your result is consistent with the transformation properties of (i) a covector x_i (the case $n = 1$), and (ii) the covariant components of the metric tensor g_{ij}.

(4 points)

1.5.2. **Curl and divergence**

Show that the curl and the divergence of a vector field transform as a pseudovector field and a scalar field, respectively.

(3 points)

1.5.3. **Tensor products, and tensor traces**

Prove Propositions 1 and 2 from ch. 1 §5.3.

(4 points)

2.2.1. **Lindhard function**

Consider the function $f : \mathbb{C} \to \mathbb{C}$ (which plays an important role in the theory of many-electron systems) defined by

$$f(z) = \log\left(\frac{z-1}{z+1}\right)$$

The *spectrum* $f'' : \mathbb{R} \to \mathbb{R}$ and the *reactive part* $f' : \mathbb{R} \to \mathbb{R}$ of f are defined by

$$f''(\omega) := \frac{1}{2i}\left[f(\omega + i0) - f(\omega - i0)\right] \qquad , \qquad f'(\omega) := \frac{1}{2}\left[f(\omega + i0) + f(\omega - i0)\right]$$

where $f(\omega \pm i0) := \lim_{\epsilon \to 0} f(\omega \pm i\epsilon)$.

a) Show that f' and f'' are indeed real-valued functions.

b) Determine f'' and f' explicitly, and plot them for $-3 < \omega < 3$.

c) Show that

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\, \frac{f''(\omega)}{\omega - z} = f(z)$$

(5 points)