PHYS 622 Classical Electrodynamics W 2021

Final Exam 03/18/2021

This is an open-book exam. You can use any inanimate resources you want, but please do not consult any live resources.

Treat this exam exactly as you would treat an in-class exam: Once you open it, you can work on it for up to two contiguous hours. Do NOT stop and resume later. You can choose any two-hour period you like, but you must submit your work by 5pm PDT on Thu, March 18, 2021 at the latest. Submit a legible scan with your name on it to dbelitz@uoregon.edu, NOT to your grader.

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1. **Magnetic Monopoles**

In ch. 1 we noticed that the Maxwell equations are weirdly asymmetrical. This can be fixed as follows. Suppose nature had decided that one 4-vector potential A^μ and one 4-current J^μ was not enough, and there was another 4-vector field $\tilde{A}^\mu$ and another 4-current $\tilde{J}^\mu$. Now define a modified field tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \partial_\kappa \epsilon_{\mu\nu}{}^{\kappa\lambda} \tilde{A}_\lambda$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the 4-dimensional Levi-Civita symbol, and a modified Lagrangian density

$$\mathcal{L} = \frac{-1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} A_\mu J^\mu - \frac{1}{c} \tilde{A}_\mu \tilde{J}^\mu$$

a) Show that the resulting Euler-Lagrange equations have the form

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu \qquad , \qquad \partial_\mu \tilde{F}^{\mu\nu} = \frac{4\pi}{c} \tilde{J}^\nu$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ is the dual field tensor from Problem 1.1.1. Compare these equations of motion with the corresponding ones in Maxwell theory.

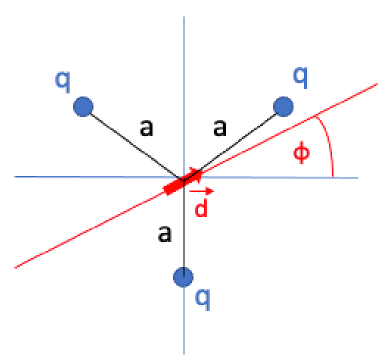
b) Define electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ by parameterizing the field tensor $F^{\mu\nu}$ as in Maxwell theory, express the equations of motion from part a) in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$, and compare the result with the usual Maxwell equations.

c) Show that the 4-current $\tilde{J}^\mu$ obeys a continuity equation and briefly discuss its physical meaning.

(8 points)

2. **Dipole in an external field**

Three equal charges q are sitting on the corners of an equilateral triangle in the x-y plane, each at a distance a from the origin. At the center of the triangle sits an electric dipole $\boldsymbol{d}$. The positions of the charges and the dipole are fixed, but the dipole is free to rotate in the x-y plane. Calculate the electrostatic interaction energy of this system to dipolar order, and determine the equilibrium orientation of the dipole (i.e., the angle ϕ that minimizes the energy).



(5 points)