PHYS 623 Classical Electrodynamics S 2021

Final Exam 06/7/2020

Instructions: Treat this exam as you would an open-book 3-hour in-class exam. You can work on it during any contiguous 3-hour period of your choosing, but once you open the exam you must keep going; you must not stop and finish it later. Scan and email your solution to dbelitz@uoregon.edu when you are done, but no later than 5pm on Tuesday, June 8. Do not discuss the exam with anyone before then.

Note: A lot of points go into the discussion of the results.

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1) Bremsstrahlung

As a simple model for an x-ray machine, consider a charged point particle that travels with constant velocity v_0 for times $t < 0$, then gets decelerated at a constant rate $-a$ (with $a > 0$ an acceleration) for times $t > 0$ until it comes to a stop at time $t = T$.

a) Calculate, sketch, and discuss the radiated energy per frequency,

$$\frac{dU}{d\omega} = \frac{2e^2}{3\pi c^3}\,|\dot{v}(\omega)|^2$$

Which frequency range dominates the spectrum? What would you do to produce radiation with higher frequency?

b) Calculate the total radiated power $\mathcal{P}$ (keeping a fixed), and show that you recover the Larmor formula

$$\mathcal{P} = \frac{2e^2}{3c^3}\,a^2$$

for a uniformly accelerated point particle.

hint: $\int_0^\infty dx\, \frac{\sin^2 x}{x^2} = \frac{\pi}{2}$

(15 points)

2) Pulsar

The pulsar SGR1806-20 is observed to have a rotational period $T = 7.5\,\text{s}$ that increases at a rate $\dot{T} = 8 \times 10^{-11}$. As a very crude model of such a system, consider a rotating homogeneous neutron star with mass $M = 3 \times 10^{33}\,\text{g}$ and radius $R = 10^6\,\text{cm}$ that rotates about the z-axis and has a magnetic moment $\boldsymbol{m}$ that lies in the $xy-$plane and rotates with the same period as the star as a whole.

a) Assuming that the increase of the period is due to magnetic dipole radiation, express the magnitude m of the dipole moment in terms of the parameters given above.

hint: Remember that the rotational kinetic energy of a homogeneous sphere is $U = I\Omega^2/2$, with Ω the angular velocity and $I = 2MR^2/5$ the moment of inertia of the sphere.

b) From the expression for the magnetic field generated by a magnetic dipole moment that we derived in ch.2 §3.7 corollary 1, estimate the maximum magnetic field at the surface of the neutron star. (Take $r = R$ and don't worry about the validity of the far-field approximation.) Determine the maximum magnetic field in units of Gauss and compare with the field the surface of the earth ($\approx 0.5\,\text{G}$).

(9 points)